Mid-West University

Examinations Management Office

End-Semester Examinations -2080

Bachelor level / B.E. Civil / 2nd Semester — Full Marks: 50

Time: 3 hours — Pass Marks: 25

Subject: Engineering Mathematics–II (SH421/SH102)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a. (i) If $u = tan(y + ax) - (y - ax)^{3/2}$, then show that ; $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}$.

   (ii) Find the volume of the solid bounded by the surface $z = 0,\ x^2 + y^2 = 1,\ x + y + z = 3.$ **(2+3)**

   b. (i) Find the minimum value of $x^2 + xy + y^2 + 3z^2$ subject to the condition $x + 2y + 4z = 60$.

   (ii) Evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dxdy$ by changing the order of the integration. **(2+3)**

2. a. (i) Find the equation of the line through $(1,2,-1)$ and perpendicular to the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

   (ii) Define the convergence and divergence of the series. Find the interval and radius of the convergence of the power series $1 + \frac{1}{5}x + \frac{2}{5^2}x^2 + \frac{3}{5^3}x^3 + \cdots$ ……… . **(2+3)**

   b. Defined the right circular cylinder. Find the equation of right circular cylinder having for its base the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3.$ **(5)**

3. a. (i) Define directional derivative of a function f in the direction of $\vec{a}$. Find the directional derivative of a function $f = x^2 - y^2 + 2z^2$ at the point A(1, 2, 3) in the direction of $\vec{a} = \vec{i} + \vec{j} + \vec{k}$.

   (ii) Test the series for convergence by root test $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$. **(2+3)**

   b. Proved that the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = x - 3y + 2z - 3 = 0$ are coplanar. Also find their point of intersection and equation of plane in which they lie. **(5)**

4. a. (i) If $\vec{r} = \vec{a}e^{nt} + \vec{b}e^{-nt}$, where $\vec{a}$ & $\vec{b}$ are constant vectors, show that: $\frac{d^2\vec{r}}{dt^2} - n^2\vec{r} = 0.$

   (ii) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$. **(2+3)**

   b. Define the Bernoulli's differential equation. Solve ; $\frac{dy}{dx} + \frac{x}{1-x^2} y = x\sqrt{y}.$ **(5)**

5. a. Defined the Legendre's polynomial. Prove that $P_n(x) = \sum_{r=0}^{n} \frac{(-1)^r (2n-r)! x^{n-2r}}{2^n r!(n-r)!(n-2r)!}$ **(1+4)**

   b. (i) If $\frac{d^2\vec{r}}{dt^2} = 6t\vec{i} - 12t^2\vec{j} + \vec{k}$ then show that: $\vec{r}'(0) = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{r}(0) = 7\vec{i} + \vec{j}$, find $\vec{r}$.

   (ii) Solve; $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10sinx.$ **(2+3)**

The End