Mid-West University

Examinations Management Office

End-Semester Examinations -2080

Bachelor level/ B.E. Hydropower/ II Semester — Full Marks: 50
Time: 4 hours — Pass Marks: 25
Subject: Engineering Mathematics-II (SH421/SH102/SH103)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a. (i) Verify the Euler's theorem for the given function : $u = sin^{-1}\left(\frac{x}{y}\right) + tan^{-1}\left(\frac{y}{x}\right)$.

(ii) Find the maximum value of xyz under the condition $x + y + z = 4$. **(2+2)**

b. (i) Evaluate the integral by changing the order of the integration; $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$.

(ii) Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the region V bounded by the planes $x = 0, y = 0, z = 0,$

$x + y + x = a$. **(3+3)**

2. a. (i) Find the equation of plane through the points (-1,2,6) and (4, -5,6) parallel to x-axis.

(ii) Define S.D. Find the magnitude and the equation of the line of S.D. between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ & $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.0 **(2+5)**

b. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9, x - 2y + 2z = 5$ as a great circle. Determine its centre and radius. **(5)**

3. a. (i) Solve for $\vec{r}$ to the equation $\frac{d^2\vec{r}}{dt^2} = 3t\vec{i} - 6t\vec{j} + \vec{k}$, Given that $\frac{d\vec{r}}{dt} = \vec{i} + 4\vec{j} + \vec{k}$ at $t = 0$

and $\vec{r} = 0$ at $t = 0$.

(ii) Define divergence and curl of the vector product. Find the unit normal to the surface

$x^3 + y^3 + 3xyz = 3$ at the point $(1,2,-1)$. **(3+3)**

b. (i) State the root test theorem. Test the following series convergence by applying root

test. $\sum\left(\frac{nx}{1+n}\right)^n$.

(ii) Find the radius and interval of convergence of the power series. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n\, 2^n}$.

(3+3)

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4. a. Solve the following differential equation. $xdy - ydx = \sqrt{x^2 + y^2}\, dx.$ (3)

   b. Solve the second order linear differential equation ; $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = sin2x.$ (3)

5. a. (i) Proved that the given Legendre's function:$x^5 = \frac{8}{63}[P_5(x) + \frac{7}{2}P_3(x) + \frac{27}{8}P_1(x)]$. (2+3)

   (ii) Proved that $\frac{d}{dx}[x^{\vartheta} j_{\vartheta}(x)] = x^{\vartheta} j_{\vartheta-1}(x)$.

   b. Defined the linear differential equation ; solve : $\frac{dy}{dx} + \frac{y}{x} = x^2$ *if* $y = 1$ *when* $x = 1$. (5)

The End