

Mid-West University
Examinations Management Office
Final Examinations-2082

Bachelor level/Environment Science/I Semester
Time: 3 hrs.
Subject: Mathematics of Environment system (ENV412)

Full Marks: 60
Pass Marks: 30

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Very short Answer Questions (Attempt any ten)

[10 × 2 = 20]

1. Define factorial of n. In how many ways can the letters of the word 'ENVIRONMENT' be arranged? [2]
2. Evaluate: $\lim_{n \rightarrow 3} \left(\frac{1}{x-3} - \frac{9}{x^3-3x^2} \right)$. [2]
3. If (3,5) and (5,7) are the points on the plane. Express \overrightarrow{AB} in the form $x\vec{i} + y\vec{j}$ and find the unit vector along \overrightarrow{AB} . [2]
4. In which condition the function has inverse. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ define by $f(x) = 5 - 4x$ find the formula for f^{-1} . [2]
5. State chain rule of derivatives. Find the derivatives of x^9 w.r.t x^4 . [2]
6. Is matrix multiplication satisfied the commutative properties? verify your answer. [0.5+1.5]
7. Evaluate: $\int \frac{x-5}{x+5} dx$. [2]
8. Define increasing and decreasing function by the help of derivatives. Show that the function $f(x) = 2x^3 - 4x^2 + 3$ is increasing at $x = 2$. [1+1]
9. Find the derivatives of the function: $\sqrt{x}(x^2 - 1)$ with respect to x. [2]
10. Determine whether the vectors; $\vec{i} - 3\vec{j} + 2\vec{k}$, $2\vec{i} - 4\vec{j} - \vec{k}$ and $3\vec{i} + 2\vec{j} - 2\vec{k}$ are linearly dependent and independent. [2]
11. Evaluate $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}} dx$. [2]
12. Find $\frac{dy}{dx}$ if $y = 3t^2$ and $x = 4t$ [2]

Group B

Short Answer Questions (Attempt Any Four)

[5 x 4 =20]

13. a) Differentiate between permutation and combination. In a group of 10 questions in two group, the first group contain 4 questions. In how many ways to solve question if at least three questions of group first must be solved? [2+2]
- b) Define continuity of a function at point. [1]

14. State any three types of discontinuity at a point. A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 3 + 2x & \text{for } -\frac{3}{2} \leq x < 0 \\ 3 - 2x & \text{for } 0 \leq x < \frac{3}{2} \\ -3 - 2x & \text{for } x \geq \frac{3}{2} \end{cases}$$

test the continuity at $x=0$.

[2+3]

15. Define injective and surjective function. Let a function $f: A \rightarrow B$ be define by $f(x) = \frac{x+1}{2x-1}$ with $A = \{-1, 0, 1, 2, 3, 4\}$ and $B = \{-1, 0, \frac{4}{5}, \frac{5}{7}, 1, 2, 3\}$. Find the range of f . Is the Function one to one and onto both? if not what the function be made one to one and onto both? [2+2+1]

16. What is the relation between derivatives and antiderivatives Evaluate the following integrals:

$$\int_0^1 e^{\sqrt{x}} dx.$$

[1+4]

17. Define symmetric and skew symmetric matrix. Express the matrix $A = \begin{bmatrix} -2 & 6 & 1 \\ 2 & 2 & 4 \\ 3 & 4 & 0 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. [2+3]

Group C

Long Answer Question (Attempt Any Two)

[2x 10 =20]

18. a) If $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ verify that $AA^{-1} = A^{-1}A = I$. [4]

b) Solve following system of linear equation by crammer's rule. [4]

$$\begin{aligned} x + 3y - z &= -2 \\ 3x + 2y - z &= 3 \\ -6x - 4y - 2z &= 18 \end{aligned}$$

c) Is the matrix multiplication satisfied commutative properties? Verify your answer. [2]

19. a) Define implicit function. Use the implicit differentiation obtain $\frac{dy}{dx}$ of $x^2 + 2x^2y = y^3$ [1+4]

b) Evaluate: $\int x^2 e^x dx$ [2]

c) obtained the area bounded by the curves $y = x^2$ and $y = 2x$ [3]

20. a) Whether the vectors $5\vec{i} + 6\vec{j} + 7\vec{k}$, $3\vec{i} + 20\vec{j} + 5\vec{k}$ and $7\vec{i} - 8\vec{j} + 9\vec{k}$ are coplanar. [3]

b) Evaluate: i. $\lim_{x \rightarrow a} \frac{\sqrt{2x} - \sqrt{3x-a}}{\sqrt{x} - \sqrt{a}}$ ii. $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$ [3]

c) Write the condition that the function $y = g(x)$ is concave upward and concave downward. [2]

d) Define the condition for parallel vector. For what value m the vectors $3\vec{i} - 4\vec{j} + m\vec{k}$ and $6\vec{i} - 8\vec{j} - \vec{k}$ are parallel. [2]

The End