Mid-West University

Examinations Management Office

End Semester Exam-2082

Level: B.Ed. / V Semester FM: 60

Time: 3 hrs PM: 30

Sub: Basic Abstract Algebra (MATH453)

Candidates are requested to give their answers in their own words as far as practicable.

Attempt All the Questions.

Group 'B' $6 \times 5 = 30$

1. Define subgroup with example. Also show that the set {1, -1} is a subgroup of a group {1, -1, i, -i} with multiplication operation.

2. Prove that, every group has a unique identity element.

3. State and prove Lagrange's Theorem.

   Or

   Show that if every element of the group G is its own inverse, then G is abelian.

4. If R and S are two subgroup of a group G, then show that $R \cap S$ are also subgroup.

5. If $(G, *)$ is a group and H is a non-empty subset of G. Then $(H, *)$ is a subgroup of G if and only if $a * b^{-1} \in H, for\ all\ a, b \in H$.

6. A non-empty subset S of a ring R is a subring of R if and only if $a, b \in S$ implies $(i)\ a - b \in S \quad (ii)\ ab \in S$.

   Or

   Define:
   a) Left ideal and right ideal.
   b) Prime ideal and maximal ideal.
   c) Ring with zero divisors.

Group 'C' $2 \times 10 = 20$

7. (a) Let $\emptyset: G \to \overline{G}$ be a group homomorphism of G onto $\overline{G}$. If K is the kernel of $\emptyset$ then prove that: $\overline{G} \cong \frac{G}{K}$.

   (b) If $\emptyset: R \to \overline{R}$ is a ring homomorphism R onto $\overline{R}$, then prove that $\emptyset$ is one to one if and only if $Ker_{\emptyset} = \{0\}$.

8. Define ring with complex numbers $(C, +, .)$ and show the set of complex numbers with respect to both operations addition and multiplication forms a ring.

   Or

   (a) If $f: A \to B$ and $g: B \to C$ are both one to one and onto mapping then prove that $(gof)^{-1} = f^{-1} o\ g^{-1}$.
   (b) Prove that: Every field is integral domain.

THE END

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Roll No. ....

Group 'A' 10×1=10

Tick (✓) the Best Answer.

1. An algebraic structure to be a group if...
   a) it satisfies closure, associative and commutative.
   b) it satisfies closure, associative, commutative and existence of inverse element.
   c) it satisfies closure, associative and commutative and existence of identity element.
   d) it satisfies closure, associative, existence of identity element and existence of inverse element.

2. A number of subgroups of a group $G = \{1.\omega, \omega^2\}$ with multiplication operation...
   a) 5 b) 6
   c) 7 d) 8

3. Every .... is unique factorization domain.
   a) integral domain b) ring
   c) principle ideal domain d) commutative ring

4. Which is correct statement of Lagrange's Theorem?
   a) The order of normal subgroup of a group is divisor of the order of the group.
   b) The order of each subgroup of a group is divisor of the order of the group.
   c) The order of each finite subgroup of a group is divisor of the order of the group.
   d) The order of the left cosets and right cosets are equal.

5. Every subgroup of an abelian group is......
   a) abelian b) normal
   c) both (a) and (b) d) not group

6. Which statement is not correct for normalizer of an element of a group G ?
   a) N(a) is not a normal subgroup of G in general
   b) $N(e) = G; for\ ex = xe\ for\ all\ x \in G.$
   c) $N(a) \neq G \quad iff\ G\ is\ abelian.$
   d) All of the above.

7. The order of the dihedral group D_4 is equals to.....
   a) 4 b) 6
   c) 8 d) 10

8. If $a \equiv b(mod\ c)$ then which is the correct statement?
   a) 'b' is dividend, 'c' is remainder and 'a' is divisor.
   b) 'a' is dividend, 'c' is remainder and 'b' is divisor.
   c) 'a' is dividend, 'b' is remainder and 'c' is divisor.
   d) None of them.

9. A fuction $f(x)$ is said to be bijective function if...
   a) $f(x)$ is one to one. b) $f(x)$ is onto.
   c) (a) and (b) both. d) None of them.

10. Which is incorrect statement?
    a) The ring $(Z, +, \times)$ of integers is an integral domains.
    b) The ring $(E, +, \times)$ of even integers is an integral domains.
    c) The ring $(R, +, \times)$ of real numbers is ring.
    d) The ring $(C, +, \times)$ of complex numbers is division ring.