

Mid-West University  
Examinations Management Office  
Surkhet, Nepal  
End Semester Examination-2082

Level: B.Ed. /VI Semester

Sub: Multivariable Calculus (MATH 463)

Roll No: .....

Group 'A'

**10 × 1 = 10**

Tick (✓) the best answers:

1. The function  $f(x, y) = ax + by + c$  is called .....
  - a. Linear function
  - b. Quadratic function
  - c. Cubic function
  - d. Bi-quadratic function
2. If  $f$  is continuous on the rectangle  $R = \{(x, y): a \leq x \leq b, c \leq y \leq d\}$ . Then, which of the following statement is correct?
  - a.  $\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dx dy$
  - b.  $\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx$
  - c.  $\iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dy dx$
  - d. None of above
3. Which of the following statements is not correct?
  - a. A continuous curve is simple if it has no multiple points.
  - b. A continuous curve is simple if the only has multiple points are the coincident end points.
  - c. A continuous curve is simple if it has multiple points.
  - d. A continuous curve is simple if it is simply a Jordan arc.
4. The Cartesian coordinate of the point  $(x, y)$  is
  - a.  $(r, \theta)$
  - b.  $(a, b)$
  - c.  $(\theta, r)$
  - d.  $(x, y)$

5. Which one is the spherical coordinate?

- a.  $(x, y, z)$
- b.  $(r, \theta)$
- c.  $(r, \rho, \theta)$
- d.  $(\rho, \theta, \varphi)$

6. A point is said to be critical or stationary point of function if...

- a.  $f_x(a, b) = 0$
- b.  $f_y(a, b) = f_x(a, b)$
- c.  $f_y(a, b) = 0$
- d. All of above

7. If  $f(x, y) = \sin x + e^{xy}$ , then the value of  $\nabla f(0, 1)$  is

- a.  $\langle 0, 1 \rangle$
- b.  $\langle 1, 0 \rangle$
- c.  $\langle 2, 0 \rangle$
- d.  $\langle 0, 2 \rangle$

8. The level of curves for a function  $z = f(x, y)$  of two variables are curves in the xy plane defined by  $f(x, y) = k$ , where

- a.  $k$  is any constant in the domain of  $f$ .
- b.  $k$  is any constant in the range of  $f$ .
- c.  $k$  is any constant of  $f$ .
- d.  $k$  is any identity in  $f$ .

9. Which of the following is an elliptic cylinder?

- a.  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$
- b.  $\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$
- c.  $\frac{x^2}{a^2} - py = 0$
- d.  $\frac{x^2}{a^2} + py = 0$

10. The function  $A = f(l, b)$  represents function of area of rectangle through

- a. Verbally
- b. Numerically
- c. Algebraically
- d. IPO

Mid-West University  
Examinations Management Office  
Surkhet, Nepal  
End Semester Examination-2082

Level: B.Ed. / VI Semester

Time: 3.00 hrs.

FM: 60

PM: 30

Sub: Multivariable Calculus (MATH 463)

*Candidates are required to give their answers in their own words as far as practicable.*

Attempt All the Questions.

Group 'B'

6×5 = 30

1. Sketch the graph of the function  $g(x, y) = \sqrt{9 - x^2 - y^2}$ . Also find the domain and range of such function.
2. Define limit of function  $f(x, y)$  at  $(a, b)$ . Find  $\lim_{(x, y) \rightarrow (0, 0)} \frac{3x^2}{x^2 + y^2}$ , if exists.
3. The dimension of a rectangular box are measured to be 67cm, 60cm and 40cm. Each measurement is correct within 0.2cm. Use differentials to estimate the largest possible error when the volume of the box is calculated from these measurements.

Or

Define implicit function. Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  if  $x^3 + y^3 + z^3 + 6xyz = 1$ .

4. Evaluate  $\iint_R (3x + 4y^2) dA$ , where  $R$  is the region in the upper half-plane bounded by the circle  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .
5. Define conservative vector field. Show that  $F(x, y, z) = y^2 z^3 \vec{i} + xyz^3 \vec{j} + xy^2 z^2 \vec{k}$  is a conservative vector field.

6. Define surface integral of type second. Evaluate  $I = \iint_S (xdydz + dzdx + xz^2 dxdy)$ , where  $S$  is the outer side of the part of the sphere  $x^2 + y^2 + z^2 = 1$  in the first octant.

Or

Find the tangent plane to the surface with parametric equations  $x = u^2$ ,  $y = v^2$  and  $z = u + 2v$  at the point  $(1, 1, 3)$ .

Group 'C'

2×10 = 20

7. Prove that if  $f$  is a differentiable function of  $x$  and  $y$ , then  $f$  has a directional derivative in the direction of unit vector  $\vec{u} = \langle a, b \rangle$  and  $D_{\vec{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b$ . Also, find the directional derivative of function  $f(x, y) = x^2 y^3 - 4y$  at the point  $(2, -1)$  in the direction of  $\vec{v} = 2\vec{i} + 5\vec{j}$ .
8. Let  $C$  be a positively oriented, piecewise smooth, simple closed curve in the plane and let  $D$  be the region bounded by  $C$ . If  $P$  and  $Q$  have continuous partial derivatives on an open region that contains  $D$ , then prove that

$$\int_C Pdx + Qdy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Or

Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder  $x = y^2$  and the planes  $x = z$ ,  $z = 0$  and  $x = 1$ .

THE END