Mid-West University

Examinations Management Office

Surkhet, Nepal

End Semester Examination-2082

Level: M.Ed. / II Semester

Sub: Linear Algebra (MATH 525)

Roll No. ....

Group "A" $10 \times 1 = 10$

Tick (✓) the best answer.

1. A non-empty subset W of vector space V is called subspace if ...
   a. v-w ∈ W $\forall v, w \in W$
   b. v+ w ∈ W $\forall v, w \in W$
   c. v-w ∈ W and c v ∈ W $\forall v, w \in W$ and c ∈ K
   d. All of the above

2. A set of vectors V= $\{v_1, v_2, v_3, \ldots, v_n\}$ is called orthogonal if
   a. $v_i . v_j = 0$ for i=j
   b. $v_i . v_j = 0$ for $i \neq j$
   c. $v_i . v_j = 1$ for i=j
   d. $v_i . v_j = 1$ for $i \neq j$

3. A symmetric bilinear form always represents the
   a. Square matrix
   b. Symmetric matrix
   c. Singular matrix
   d. Triangular matrix

4. A bilinear form g: V× V→ K is called skew symmetric if
   a. g (v, w)= - g(w ,v)
   b. g (v, w)= g(w ,v)
   c. g (v, w)= 0
   d. g (v, w)= identity

5. Let ϕ: V →W be a line map. Then (Ker ϕ) is ...
   a. subspace of V
   b. subspace of W
   c. subset of image ϕ
   d. subspace of both V and W

6. Let V be a finite dimensional vector space over field K and A: V →V be an operator. Then for λ ∈K, v ∈V is called eigenvector of A if ...
   a. Av= A λ
   b. V= A λ
   c. Av= λ
   d. Av= λv

7. The matrix $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ is ...
   a. singular matrix
   b. symmetric matrix
   c. triangular matrix
   d. diagonal matrix

8. A set of vectors $\{v_1, v_2, \ldots v_n\}$ is said to be the basis of V if ...
   a. it generates V
   b. it is linearly independent
   c. it linearly dependent
   d. a and b both

9. Every unitary free module is ...
   a. injective module
   b. torsion module
   c. projective module
   d. quotient module

10. If M is an R-module then which of the following is not true?
    a. $r.0_M = 0_M$
    b. $0_R .m = 0_M$
    c. (–r) m= -(r m) = r (-m)
    d. $0_R.m = 0_R$

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Level: M.Ed. / II Semester

Time: 3.00 hrs.

FM: 60

PM: 30

Sub: Linear Algebra (MATH 525)

Candidates are required to give their answers in their own words as far as practicable.

Attempt all the questions.

Group "B" $6\times5=30$

1. Define linear map, its kernel and image. Prove that the composition of two linear maps is also linear.

2. Define quadratic form and find the quadratic form associated with the matrix C= $\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix}$

3. State and prove Spectral theorem.

Or

State and prove Sylvester theorem.

4. Find the eigenvalue and corresponding eigenvectors of the matrix A= $\begin{pmatrix}2 & 4\\ 5 & 3\end{pmatrix}$

5. Prove that a matrix always represents a symmetric bilinear form if and only if it is symmetric matrix.

6. Show that the map f: $Z\to Z$ by $f(x) = 2x$ is module homomorphism but not a ring homomorphism.

Or

Define the terms: Module, module homomorphism, exact sequence, free module and torsion module.

Group "C" $2\times10=20$

7. Let $0\to A_1\to B \to A_2 \to 0$ be short exact sequence of R-module homomorphisms f and g. Then the following conditions are equivalent:
   i. There exists an R-module homomorphism k: $B \to A_1$ with k f=I A_1
   ii. There exists an R-module homomorphism h: $A_2 \to B$ with g h=IA_2
   iii. The given sequence is isomorphic to the sequence $0\to A_1\to A_1 \oplus A_2 \to A_2 \to 0$

8. Define eigenvalue and eigenvector. Let V be finite dimensional vector space over field K and let A: $V\to V$ be an operator. Let $v_1, v_2, \ldots, v_m$ be eigenvectors of A with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_m$ respectively. Assume that these vectors are distinct ($\lambda_i \neq \lambda_j$). Then show that $v_1, v_2, \ldots, v_m$ are linearly independent

Or

State and prove primary decomposition theorem.

THE END