Candidates are required to give answers in their own words as far as practicable. Figures in the margin indicate full marks.

Group A   (4 x 6 = 24)

1. Define conditionally convergent. Show that if a series $\sum a_n$ is absolutely convergent, then it is convergent. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum $\sum_{n-1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

2. Find the Maclaurin series for $\cos x$. Prove that e^x is equal to the sum of its Maclaurin series.

3. A polar equation of conic of the form $r = \frac{ed}{1 \pm e \cos \theta}$ $r = \frac{ed}{1 \pm e \sin \theta}$ where the conic the conic represents an ellipse if e < 1, a parabola if e = 1 or hyperbola if e > 1.

OR

Solve the initial-value problem
$$y'' - 2y' + 5y = 0, \; y(\pi) = 0, \; y'(\pi) = 2$$

4. Find the vector equation and parametric equation for line that passes through the point (5, 1, 3) and is parallel to the vector $\vec{i} + 4\vec{j} - 2\vec{k}$. Also fine parametric equations and symmetric equation of the line that passes through the point A(2,4,-3), and B(3,-1, 1).

Group B (6 x 4 = 24)

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward?
$$x = 4 + t^2, \; y = t^2 + t^3$$

6. Sketch and identify the curve defined by the parametric equations $x = t^2 - 2t, \; y = t + 1$

7. Solve the differential equation $y'' + 9y = e^{3x}$

8. If $\vec{r} = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$ where f, g and h are differentiable functions then If
$$\vec{r} = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\vec{i} + g'(t)\vec{j} + h'(t)\vec{k}$$

9. If the series $\sum_{n=1}^{\infty} a_n$ is convergent then $\lim_{n \to \infty} a_n = 0$.

Determine whether the given series $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \cdots$ convergent or not. If convergent fine the sum.

OR

Suppose that $\sum a_n$ and $\sum b_n$ are the series with positive terms.

a. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n then $\sum a_n$ is also convergent.

b. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n then $\sum a_n$ is also divergent

10. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

Group C [3 (2+ 2) = 12]

11. a. Express the parametric equations $x = 3 \sin \theta$ and $y = 4 \cos \theta$ in Cartesian form.

b. Find the sum of the series $\sum_{n=0}^{\infty} x^n$, where $|x| < 1$.

12. a. Determine whether the series converges or diverges

$$\sum_{m=1}^{\infty} \frac{n}{2n^3 + 1}$$

b. If $\vec{a}$ $\vec{b}$ and $\vec{c}$ be three vectors in V_3 then prove that

$$\vec{a}.(\vec{b}+ \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$$

13. a. Find the domain of the vector function

$$\vec{r} = \frac{t-2}{t+2}\vec{\imath} + \sin t\,\vec{\jmath} + \ln(9 - t^2)\vec{k}$$

b. Solve the differential equation $y'' - 8y' + 12y = 0$

The End