Mid-West University

Examinations Management Office

Surkhet, Nepal

End Semester Examination 2080

Level: B.Ed. / VI Semester FM: 60

Time: 3 hrs PM: 30

Sub: Abstract Algebra (Math 465)

Candidates are requested to give their answers in their own words as far as practicable.

Attempt All the Questions.

Group 'B' **6 × 5 = 30**

1. Show that the order of a cyclic group is the same as the order of its generator.

2. Prove that: the set of all permutations on n symbols forms a group under composition of permutation.

3. If G_1 and G_2 be two abelian groups then prove that $G_1 \times G_2$ is an abelian group.

   Or

   Define refinement of a series. Prove that: every refinement of a solvable series is solvable.

4. Show that: every finite extension K of a field F is algebraic.

5. Write the difference between integral domain, division ring and field with examples.

6. Show that: every finite p-group G is solvable.

   Or

   Define internal direct product and external direct product with examples.

Group 'C' **2 × 10=20**

7. Show that: that the ring $z[x]$, the ring of all polynomials with integer coefficients, is an integral domain.

8. (a) Define principal ideal domain and unique factorization domain.

   (b) Prove that: every principal ideal domain is a unique factorization domain.

   Or

   Let K be a field. Then the following are equivalent:

   (a) K is algebraically closed.

   (b) Every irreducible polynomial in K[x] is of degree 1.

   (c) Every polynomial in K[x] of positive degree factors completely in K[x] into linear factors.

   (d) Every polynomial in K[x] of positive degree has at least one root in K.

THE END

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Roll No. .....

Group 'A' **10×1=10**

Tick (✓) the Best Answer.

1. The order of 'w' in cyclic group $G = \{1, w, w^2\}$, where 'w' is the cube root of unity, is equal to .....
   a. 1 b. 2
   c. 3 d. 4

2. Which one of the followings is true?
   a. Group of order 31 is abelian
   b. Every group of order 4 is cyclic.
   c. Every group of order 6 is abelian.
   d. There does not exists abelian group of order 2021.

3. If G is a prime order of group then G has.....
   a. No proper subgroup
   b. No improper subgroup
   c. Two improper subgroup
   d. None of these

4. The number of proper subgroup of Z_6 is.....
   a. 1 b. 2
   c. 3 d. 4

5. The number of solution of the equation $x^2 = 1$ in the ring $\frac{Z}{30Z}$
   a. 4 b. 6
   c. 8 d. 10

6. Let F be a field with 3^{10} elements. What is the total number of proper subfield of F?
   a. 2 b. 3
   c. 4 d. 5

7. Let R be a ring of all matrices of order 2×2 over integers and S be the set of 2×2 matricesof the form $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$ where a, b are integers then
   a. S is the left ideal
   b. S is the right ideal
   c. S is the left and right ideal both
   d. None of the above

8. The splitting field of $f(x) = x^4 - 5x^2 + 6 \in Q[x]$ is
   a. $Q(\sqrt{2})$ $Q(\sqrt{3})$
   c. $Q(\sqrt{2}, \sqrt{3})$ All of above

9. If U an ideal of ring R and $1 \in U$
   a. U is proper subset of R
   b. $U = R$
   c. $U = \emptyset$
   d. U is super set of R

10. An element 'a' in a euclidean domain is a unit.
    a. If $d(a) = 1$
    b. iff $d(a) = d(1)$
    c. If b is unit in R, then $d(ab) = d(a)$
    d. None of these.