Mid-West University

Examinations Management Office

End Semester Exam-2081

B.Ed. Level /V Semester

Sub: Basic Abstract Algebra (MATH453)

Roll No. ....

Group 'A' **10×1=10**

Tick (✓) the Best Answer.

1. Let $G = \{a: a^5 = e \text{ (identity)}\}$. Then the elements of G are:
   a. $\{a, a^2, a^3, a^4\}$ b. $\{a^2, a^3, a^4\}$
   c. $\{a^3, a^4, a^5\}$ d. $\{e, a, a^2, a^3, a^4\}$

2. The set of integers Z with the binary operation "*" defined as a*b =a +b for a, b ∈ Z, is a group. The identity element of this group is
   a. 0 b. 1
   c. -1 d. 12

3. Let G be a group where * as a usual operation. For a, b ∈ G, then which is correct
   a. $(a^{-1})^{-1} = a^{-1}$ b. $(a * b)^{-1} = a^{-1} * b^{-1}$
   b. $(a * b)^{-1} = b^{-1} * a^{-1}$ d. All of the above.

4. The order of symmetric group S_3 is ...
   a. 13 b. 6
   b. 4 c. 3

5. Which one if the followings is not correct?
   a. The set of integers with respect to addition forms a group.
   b. The set of integers (Z, +) is a commutative group.
   c. The set of rational numbers (Q, +) is a group.
   d. The set of odd integers is group under 't'.

6. Let A be a commutative ring with unity with two binary operations addition and multiplication, then A can be
   a. An empty set.
   b. {2}.
   c. {0}.
   d. Set of all natural numbers

7. Let A be a commutative ring with unity. Then,
   a. Every unit element is a zerodivisor.
   b. Every nilpotent element is a zerodivisor.
   c. Every zerodivisor is a unit element.
   d. Every zerodivisor is a nilpotent element.

8. How many subgroups can be formed from S_3= {(1), (12), (13), (23), (123), (132)}?
   a. 4 b. 5
   c. 6 d. 3

9. Let a and b be two ideals of a commutative ring A with unity. Then which of the following is falls?
   a. a ∩ b is an ideal in A. b. ab is an ideal in A.
   c. a + b is an ideal in A. d. a ∪ b is an ideal in A.

10. Let q be an ideal in a ring A,
    a. If q is primary, then A q = 0.
    b. If A q = 0 then q is not primary.
    c. If q is primary, then there exists a zero-divisor in A q which is not nilpotent.
    d. There exists a prime ideal which is not primary.

Mid-West University

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End Semester Exam-2081

Level: B.Ed. / V Semester FM: 60

Time: 3 hrs PM: 30

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Candidates are requested to give their answers in their own words as far as practicable.

Attempt All the Questions.

Group 'B' $6 \times 5 = 30$

1. Define group with example. Also show that the set {1, -1, i, -i} is a multiplicative finite group.

2. If (G, o) be a group and H be a non-empty subset of G. Then (H, o) is a subgroup of G iff $aob^{-1} \in H$, for all $a, b \in H$. Pove it.

3. Write the definition of,
   a) Commutative group
   b) Division ring
   c) Integral domain

   Or

   Let $f : G \to \overline{G}$ be a group homomorphism of G onto $\overline{G}$. If K is the kernel of f then then prove that: $\overline{G} \cong \frac{G}{K}$

4. State and prove Lagrange's theorem.

5. Prove that a finite integral domain is a field.

6. Prove that every subgroup of an abelian group is normal.

   Or

   A non-empty subset S of a ring R is a subring of R if and only if $a, b \in S$ implies (i) $a - b \in S$ (ii) $ab \in S$.

Group 'C' $2 \times 10 = 20$

7. Let R be a ring and I be an ideal in R. Then the set $\frac{R}{I} = \{a + I : a \in R\}$ in which two operations addition and multiplication are defined as $(a + I) + (b + I) = (a + b) + I$ and $(a + I)(b + I) = (ab) + I$ for all $a, b \in R$ forms a ring.

8. Show that the set of real number with two binary operations addition and multiplication $(R, +, .)$ forms a ring with showing all properties of ring.

   Or

   Let $(G, *)$ be a group. If H and K are non- empty finite subset of G, then
   a) $(H, *)$ is a subgroup of G if and only if $a * b \in H$, for all $a, b \in H$.
   b) If H and K are subgroups of group G, then $H \cap K$ is also a subgroup

THE END