Mid-Western University

Examinations Management Office

Chance Examinations -2081

Bachelor level/ B.Sc. / 8th Semester — F.M.: 100

Time: 3hrs — P.M.: 50

Subject: Mathematical Physics (PHY483)

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

1. Attempt **ANY EIGHT** Questions **[8×2=16]**

a. State the physical meaning of a gradient of a scalar function.

b. If $\vec{v} = \vec{\omega} + \vec{r}$, prove that $\vec{\omega} = \frac{1}{2}\vec{v}$ where, ω is constant vectors.

c. Distinguish between a symmetrical and skew-symmetrical tensor.

d. Show that any inner product of the tensors A_t^p and B_t^q is a tensor of rank two outer product of A_t^p and $A_t^q = A_t^p B_t^q$.

e. Prove that δ_q^p is a mixed tensor of the second rank.

f. Prove that $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a Unitary matrix.

g. Find out the Laplace transform of e^{sa}.

h. Find out the Fourier transform of the second derivative of the function $f(x)$.

i. Write the Bessel differential equation. With its physical significance.

j. Eigenvalues are invariant under a similarity transformation.

Group B

2. Attempt **ANY SIX** Questions **[6×6=36]**

I. State and prove the Divergence theorem in vector analysis.

II. Derive transformation laws for the Christoffel symbols of

(a) the first kind (b) the second.

III. Find the Laplace transforms of $f(t) = sinat \; and \; f(t) = t \; cosat$.

IV. What is the Convolution theorem for Fourier transforms? Derive an expression for it.

V. Define symmetric and anti-symmetric matrices with suitable examples. Find the inverse of a matrix $A = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$.

VI. Using the generating function of Hermite polynomial, show that $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$.

VII. Show that Legendre's polynomials are a set of orthogonal functions in the interval (-1,1)

i.e. $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \; if \; m \neq n$.

Group C

3. What do you understand by a Fourier series? Hence find coefficients of Fourier sine and cosine series. How do you express in a complex form?

Or

Solve Legendre's differential equation by series solution method. Hence write the expression for associated Legendre's polynomial. **[9]**

4. Show that a symmetric tensor is symmetric in all coordinate systems. What do you understand by covariant and contravariant tensors? [3+6]

5. Show that $\nabla|\vec{r}|^m = m|\vec{r}|^{m-2}\vec{r}$ [6]

Or

Determine the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$.

6. Find the directional derivative of $\phi = x^2y + xyz$ at $(1,2,-1)$ in the direction $A = 2i - 2j + k$. [6]

1. Find Fourier series expansion for $f(x) = |sinx|$

$f(x) = \begin{cases} -sinx & for -\pi \le x < 0 \\ sinx & for 0 \le x < \pi \end{cases}$ [6]

7. Solve $y'' + 4y' + 4y = t^2$ with initial conditions $y(0) = 0$ and $y'(0) = 0$. [6]

8. Using $J_n(x) = \sum_{r=0}^{\infty}(-1)^r \frac{1}{r!(n+r)!}\left(\frac{x}{2}\right)^{n+2r}$, where n is a positive integer, show that $\frac{2n}{x}J_n(x) = \{J_{n+1}(x) + J_{n-1}(x)\}$. [6]

The end