

Mid-West University
Examinations Management Office
Chance Examinations -2080

Bachelor level/ B.Sc. / 8th Semester

Time: 3hrs

Subject: Mathematical Physics (PHY483)

Full Marks: 100

Pass Marks: 50

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tes are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

1. Attempt **any Eight** Questions. [8x2=16]

- a. Explain divergence of a vector.
- b. If $\vec{v} = \vec{\omega} + \vec{r}$, prove that $\vec{\omega} = \frac{1}{2}\vec{v}$ where, ω is constant vectors.
- c. Distinguish between a symmetrical and anti-symmetrical tensor.
- d. Show that any inner product of the tensors M_t^p and N_t^q is a tensor of rank two outer product of M_t^p and $N_t^q = M_t^p N_t^q$.
- e. Prove that δ_q^p is a mixed tensor of the second rank.
- f. Prove that $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a Unitary matrix.
- g. Find out the Laplace transform of e^a .
- h. Find out the Fourier transform of the second derivative of the function $f(x)$.
- i. Write the orthogonality of the Legendre polynomial. Give its physical significance.
- j. Eigenvalues are invariant under a similarity transformation.

Group B

Attempt **any SIX** questions. [6x6=36]

2. State and prove the Stoke's theorem in vector analysis.
3. Derive transformation laws for the Christoffel symbols of (a) the first kind, (b) the second kind.
4. Find the Laplace transforms of $f(t) = \sin at$ and $f(t) = t \cos at$.
5. What is the Convolution theorem for Fourier transforms? Derive it.
6. Define symmetric and anti-symmetric matrix with examples. Find the inverse of a matrix $A = \begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix}$.
7. Using generating function of Hermite polynomial, show that $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$.
8. Show that Legendre's polynomials are a set of orthogonal functions in the interval (-1,1) i.e.
$$\int_{-1}^1 P_m(x)P_n(x)dx = 0 \text{ if } m \neq n.$$

Group C

9. What do you understand by a Fourier series? Hence find coefficients of Fourier sine and cosine series. Can the series be expressed in a complex form? How?

Or

Solve Bessel differential equation by series solution method. Hence write an expression for the associated Bessel polynomial. [9]

10. Show that a symmetric tensor is symmetric in all coordinate systems. What do you understand by covariant and contravariant tensors? [3+6]

11. Show that $\nabla|\vec{r}|^m = m|\vec{r}|^{m-2}\vec{r}$. [6]

Or

Prove that $H_n(-x) = (-1)^n H_n(x)$.

12. Find the directional derivative of $\phi = x^2y + xyz$ at $(1,2,-1)$ in the direction $\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. [6]

13. Find Fourier series expansion for $f(x) = |x|$ $f(x) = \begin{cases} -x & \text{for } -\pi \leq x < 0 \\ x & \text{for } 0 \leq x < \pi \end{cases}$ [6]

14. Solve $y'' + 4y' + 4y = te^{-2t}$ with initial conditions $y(0) = 0$ and $y'(0) = 0$. [6]

15. Using $J_n(x) = \sum_{r=0}^{\infty} (-1)^r \frac{1}{r!(n+r)!} \left(\frac{x}{2}\right)^{n+2r}$, where n is a positive integer,

show that $\frac{2n}{x} J_n(x) = \{J_{n+1}(x) + J_{n-1}(x)\}$. [6]

The End