

Mid-West University
Examinations Management Office
Final Examinations-2080

Bachelor level/ B. Sc. /2nd Semester

Time: 3 hours

Subject: Probability Theory (STA425/STAT325)

Full Marks: 60

Pass Marks:30

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Long answer questions. (Attempt all)

(4x6=24)

1. What is Gaussian distribution? Derive mean and variance of this distribution.
2. What is statistical definition of probability? State and prove Baye's theorem for future events.
3. Prove that hyper geometric distribution tends to binomial distribution as $N \rightarrow \infty$ and $\frac{M}{N} \rightarrow p$. Find mean and variance of hyper geometric distribution.
4. In Binomial distribution prove that $\mu_{r+1} = pq \left\{ nr \mu_{r-1} + \frac{d\mu_r}{dp} \right\}$

Or

Mention conditions under which binomial distribution tends to Poisson distribution. If X is a Poisson variate with mean m, show that $\frac{X-m}{\sqrt{m}}$ is a variable with mean 0 and variance.

Group B

Descriptive answer questions (Attempt all).

(6x4=24)

5. What is characteristic function? If $y = \frac{x-a}{b}$ then prove that $\phi_y(t) = e^{-\frac{iat}{b}} \phi_x(t/b)$.
6. If a random variable X has the probability density function
$$f(x) = \begin{cases} 2kx & ; 0 < x \leq 1 \\ k(3-x) & ; 1 < x \leq 2 \\ k & ; 2 < x \leq 3 \\ k(4-x) & ; 3 < x \leq 4 \\ 0 & ; elsewhere \end{cases}$$
 - i. Determine the value of k.
 - ii. Find $P(X > 3)$, $P(|x| < 1.5)$, $P(1 < X < 3)$
 - iii. If two events A and B are; $A = -1.5 < X < 1.5$ and $B = 1 < X < 3$, find $P(A/B)$.
7. State and prove addition theorem of expectation.
8. In an IQ test score is normally distributed with the average score of 500 children was found to be 60 and standard deviation 25. Find the i) number of children exceeding the score 70. ii) number of children with score between 65 to 80.
9. Telephone calls enter a college switchboard on the average of two every 3 minutes. If one assumes an approximate Poisson process what is the probability that i) three or more calls arrive in a 9 minutes interval. ii) at least two calls arrive in 6 minutes interval iii) exactly one call arrive in 3 minutes interval iv) No calls arrive in one minute interval.

10. For n events $A_1, A_2, A_3, \dots, A_n$

$$P(\cup_{i=1}^n A_i) \leq \sum_{i=1}^n P(A_i)$$

Or

Prove that moment generating function of Binomial distribution tends to Poisson distribution as $n \rightarrow \infty$ and $np = \lambda$.

Group C

Short answer questions. (Attempt all).

(6x2=12)

11. Define negative binomial distribution.
12. Discuss the uniform distribution.
13. What is distribution function? Write down its properties.
14. Find the probability of success for the binomial distribution if $n = 6$ and $4P(x=4) = P(x=2)$.
15. Prove that $V(aX+b) = a^2V(X)$.
16. Discuss importance of normal distribution.

The End