Mid-West University

Examinations Management Office

End Semester Examination 2081

Bachelor level/ B. Sc. /5th Semester — Full Marks: 60

Time: 3 hours — Pass Marks:30

Subject: Linear Algebra-II (MTH451/MATH451)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group-A [4x6 = 24]

1. Show that the intersection of two vector subspace is also vector subspace of a vector space. Is the union of two vector subspace is also vector subspace of a vector space? justify your answer. Let S be the set of all integers. Given $a, b \epsilon S$ define $a \sim b$ if $a - b$ is an even integer and prove that this defines an equivalence relation on S.
2. Define diagonalizable. Show that the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ is diagonalizable. Define congruent Modulo n. Let $a, b, c, d \in \mathbb{Z}$ with $a \cong b (mod\ n)$ and c $\cong d (mod\ n)$, then prove that $a + c \cong b + d (mod\ n)$.

OR

Define finite and infinite Dimensional vector space. Show that two finite - Dimensional vector space are isomorphic to each other if and only if they have same dimension.

3. Show that two finite dimensional vector space are isomorphic if they are of same dimension.
4. Find the LU factorization of $A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix}$. Use this LU factorization of A to solve AX = b , where $b = \begin{bmatrix} -9 \\ 5 \\ 7 \\ 11 \end{bmatrix}$.

Group-B [6x4 = 24]

5. Find the coordinate vector $[X]_B$ of X relative to the given basis $B = \{b_1, b_2, b_3\}$ where $b_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$,$b_2 = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}$,$b_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$, $X = \begin{bmatrix} -8 \\ -9 \\ 6 \end{bmatrix}$.
6. Find the row space, column space and null space of matrix:
$A = \begin{bmatrix} 3 & 5 & 11 & 5 & 8 \\ 0 & 0 & 2 & 1 & 5 \\ 6 & 10 & 22 & 10 & 20 \end{bmatrix}$.

OR

What do you mean by row rank and column rank? Find the row rank and column rank of matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 8 \end{bmatrix}$.

7. Show that any set of eigen vector corresponding to distinct eigen vector of matrix is linearly independent.

8. Let, $\mathcal{B} = \{u_1, u_2\}$ where, $u_1 = (1,3)$, $u_2 = (-2,1)$. Let, $\mathcal{C} = \{v_1, v_2\}$ where, $v_1 = (2,7)$, $v_2 = (1,2)$. Define linear transformation T such that $T(u_1) = 2v_1 - 3v_2$ and $T(u_2) = v_1 + 4v_2$. Find the matrix A such that $[T(x)]_{\mathcal{C}} = A[x]_{\mathcal{B}}$.

9. I State the Cayley-Hamilton theorem. Find A^{-1} by using Cayley-Hamilton theorem where $A = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 1 & 7 \\ 3 & 0 & 11 \end{bmatrix}$.

10. Find the least square solution of the system:
$$\begin{aligned} x + 5y &= 4 \\ 3x + y &= -2 \\ -2x + 4y &= -3 \end{aligned}$$

Group-C [6x2 = 12]

11. Show that Kernel of linear mapping is a subspace of the domain of that mapping.

12. If for any square matrix A given that $= PDP^{-1}$, where D is diagonalizable matrix and P is interval matrix then find expression of A^k.

13. Define Mutually orthogonal vectors. Verify that the sat $U = \{(1,0,0)\,,(0,1,,0)\,,(0,0,2)\}$ in R^3 is a basis for R^3.

14. Show that $\mathbb{R}^{2\times 2}$ is isomorphic to $\mathbb{R}^4$

15. Expand $\langle 3u + 5v, 4u - 6v \rangle$ and find the value of $\|u + v\|$ where $u = (1,2,4), v = (2,-3,5)$.

16. Define Harmitianmatrix. Verify that the matrix:

$$A = \begin{bmatrix} 4 & 3+i & 1+\sqrt{5}i \\ 3-i & 6 & 4i \\ 1-\sqrt{5}i & -4i & -5 \end{bmatrix}$$

The End