Mid-West University

Examinations Management Office

End Semester Examination 2081

Bachelor level/ B. Sc. /3rd Semester — Full Marks: 60

Time: 3 hours — Pass Marks: 30

Subject: Calculus III (MTH433/MATH333)

Candidates are required to give answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Attempt all questions [4x6 = 24]

1. Use the Lagrange multiplier to find the maximum and minimum value of the functions subject to the given function $f(x, y) = 4x + 6y$; $x^2 + y^2 = 13$.

2. $\iiint_R xydV$, where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.

3. Let *C* be a positively oriented, piecewise-smooth, simple closed curve in the plane and let *D* be the region bounded by *C*. If *P* and *Q* have continuous partial derivatives on an open region that contains *D*, then prove the relation.

$$\int_C Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

4. Find the mass and center of mass of the lamina that occupies the region *D* and has the given density function ρ, where

$$D = \{(x, y) : 0 \leq x \leq 2, -1 \leq y \leq 1\}; \ \rho(x, y) = xy^2.$$

OR

Suppose **F** is a vector field that is continuous on an open connected region D. If $\int_C \boldsymbol{F}.\boldsymbol{dr}$ is independent of path in D, then **F** is a conservative vector field on D ; that is, there exists a *f* function such that $\nabla f = $ **F.**.

Group B

Attempt all questions [6x4 = 24]

5. Find the equation tangent plane to the given surface $z = 2x^2 + y^2$ at (1,1,3). If *z=f*(*x, y*) is a differential function of *x* and *y*, where *x* = *g*(*t*) and *y* = *h*(*t*) are differentiable function of *t*, then *z* is a differential function of t and $\frac{dz}{dt} = \frac{\partial f}{\partial x}.\frac{dx}{dt} + \frac{\partial f}{\partial y}.\frac{dy}{dx}$.

6. Define the triple integral in polar form. Evaluate $\iint_R (3x + 4y^2)dA$, where *R* is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

7. Show that $f(x, y) = xe^{xy}$ is the differentiable at (1, 0) and find its linearization there. Then use it to approximate $f(1.1, -0.1)$.

OR

The pressure, volume and the temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$ where P is measured in Kilopascal V in liters and T in kelvin. Use the differential to find the approximate change in the pressure if the volume increase from 12L to 12.3L and the temperature decrease from 310k to 305k.

8. Estimate the double integral $\iint_R f(x,y)dA$ with m = n = 4 by choosing the sample point to be the points farthest form the origin.

x \ y	0	1	2	3	4
1.0	**2**	**0**	**-3**	**-6**	**- 5**
1.5	**3**	**1**	**-4**	**-8**	**-6**
2.0	**4**	**3**	**0**	**-5**	**-8**
2.5	**5**	**5**	**3**	**-1**	**-4**
3.0	**7**	**8**	**6**	**3**	**0**

9. The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's Law V=I R , to find how the current I is changing at the moment when R = 400Ω, I=0.08 A, $\frac{dV}{dt} = -0.01\ V/s$ and $\frac{dR}{dt} = 0.03\ \Omega/s$.

10. Find the potential function for $\boldsymbol{F}(x,y) = \langle 2xy^3,\ 3x^2y^2 + \cos y\rangle$ thereby showing that $\boldsymbol{F}$ is conservative.

Group C

Attempt *any six* questions [6x2 = 12]

11. Find the domain of the following functions $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$.
12. Show that the function $f(x,y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x, y) approaches (0,0).
13. Calculate f_{xxyz} if $f(x,y,z) = \sin(3x + yz)$.
14. Verify the Clairaut's Theorem for $u = \ln(2x + 3y)$.
15. Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle $R = \{(x,y): 0 \le x \le 6, \quad 0 \le y \le 4\}$.
    Use the Riemann sum with m = 3, n = 2, and take the sample point to be the upper right corner of each square.
16. Evaluate the integral $\iiint_R (xy + z^2)dV$, where $E = \{(x,y,z),\ 0 \le x \le 2, 0 \le y \le 1, 0 \le z \le 3\}$.
17. Evaluate $\int_0^\infty \int_0^x x\, e^{\frac{-x^2}{y}} dxdy$ by changing the order of integration.
18. Find the divergence, and curl of the given vector point function $\vec{f} = x^2z\vec{\imath} - 2y^3z^2\vec{\jmath} + xy^2z\vec{k})$ at the point (1, -1, 1).

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