Mid-West University

Examinations Management Office

Semester End Examination 2080

Bachelor level/ B. Sc. (CSIT)/ II Semester **Full Marks: 60**

Time: 3 hours **Pass Marks: 30**

Subject: Mathematics II (MTH425)

Candidates are required to give their answer in their own words as far aspracticable. The figures in the margin indicate full marks.

Group A

Very short Answer Questions. **[4x(2+2) = 16]**

1. a) Define types of solution of system of linear equation with example. **[2]**

   b) In which value of h and k such that the augmented matrix of linear system is consistent: $\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$. **[2]**

2. a) Define linearly dependent and linearly independent of vector with example. **[2]**

   b. Define Null space. If X = $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and A = . $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, Is X $\in NulA$. **[2]**

3. a) Prove that every vector has unique additive inverse. **[2]**

   b) Is the set H of all matrices of the form $\begin{bmatrix} 2a & b \\ 3a+b & 3b \end{bmatrix}$ a subspace of $M_{2\times2}$? **[2]**

4. a) Define eigenvalue and eigenvector. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, are u and v eigen vector of A? **[1+1]**

   b) Define inner product of vectors. Let u = (3, 4) and v = (-8, 6) find the angle between two vectors u and v. **[2]**

Group B.

Short Answer Questions (Attempt any five) **[5x4 = 20]**

5. Define the linear combination of a vectors. Is the vector

   $w = (-1, 3, 7)$ a linear combination of the vectors $u = (4, 2, 7)$ and $v = (3, 1, 4)$? **[1+3]**

6. Define linear transformation.Let $T: R^2 \to R^3$ be define by

   $T(s,t) = (2s + 3t, -s + 5t, 4s - 3t)$. Show that T is linear. **[1+3]**

7. Find the inverse of block matrix = $\begin{bmatrix} 3 & 1 & . & 1 & 2 \\ 5 & 2 & : & 2 & 3 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & . & 5 & 1 \\ 0 & 0 & . & 8 & 2 \end{bmatrix}$. **[4]**

8. Find the determinant by row reduced to echelon form: $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 4 \\ 3 & -1 & -2 & 3 \end{vmatrix}$. **[4]**

9. Let $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}, b_3 = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$ & $X = \begin{bmatrix} -8 \\ 2 \\ 3 \end{bmatrix}. [X]_B$

a) Show that the set $B = \{b_1, b_2, b_3\}$ is a basis for $\boldsymbol{R^3}$. **[1]**

b) Find the change of coordinates matrix from B to standard basis. **[0.5]**

c) Write the equation that relates x in $\boldsymbol{R^3}$ **to** $[X]_B$. **[0.5]**

d) Find $[X]_B$ for the X given above. **[2]**

10. If $S = \{u_1, u_2, \ldots. u_p\}$ is a orthogonal set of nonzero vector in $\boldsymbol{R^n}$ **,** then S is linearly independent and hence is a basis for the subspace spanned by S**.** **[4]**

Group C

Long Answer Question (Attempt any three) **[3x8 = 24]**

11. What are the types of elementary row operations? Define basic variables and free variables with example? Find the general solution of system. **[1+2+5]**

$$x_1 - 2x_2 - x_3 + 3x_4 = 0$$
$$-2x_1 + 4x_2 + 5x_3 - 5x_4 = 3$$
$$3x_1 - 6x_2 - 6x_3 + 8x_4 = 2$$

12. a) Is the matrix multiplication satisfied the commutative properties? Verify your answer. **[2]**

b) Use the crammer's rule to solve the system: **[2]**

5x - 3y = 9 and 2x + 5y = 16.

c) Daigonalize the matrix: $A = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}$. **[4]**

13. Define dimension of row space the null space and the column space. Find the basis for row space the null space and the column space of $:A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & 3 \end{bmatrix}$. **[2+6]**

15. a) Find QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. **[6]**

b) Use Gram-Schmidit process to produce an orthogonal basis for W. Where $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -8 \end{bmatrix}$. **[2]**

The End