Mid-West University

Examinations Management Office

End Semester Examination 2081

Bachelor level/ B. Sc./ 7th Semester — Full Marks: 100

Time: 3 hours — Pass Marks: 50

Subject: Ordinary Differential Equation (MATH473)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A [6x(2+2) = 24]

1. **a.** Define ordinary differential equation with their properties. Also define order and degree of the differential equation.

   b. Verify that the function $y(t) = e^{-t} + \frac{t}{3}$ is a solution of the differential equation: $y'''' + 4y''' + 3y = t$.

2. **a.** Solve the differential equation: $y' + \frac{1}{2}y = \frac{1}{2}e^{t/3}$. Also write the domain of the solution.

   b. Find the solution of differential equation: $Mdx + Ndy = 0\,, if\ Mx + Ny = 0$

3. **a.** Define Wronskian determinant. Suppose that $y_1(x) = e^{r_1 x}$ and $y_2(x) = e^{r_2 x}$ are the solution of the equation $Y(x) = c_1 y_1(x) + c_2 y_2(x)$ Show that $y_1 and y_2$ are fundamental set of solution if $r_1 \neq r_2$.

   b. Define auxiliary equation. Write the nature of solution of homogeneous secondary differential equation

4. **a.** Find the particular solution of $y'' - 2y' + y = xe^x$ by the method of undetermined coefficient.

   b. Define nth order linear differential equation with constant coefficient with an example.

5. **a.** Solve the boundary value problem: $y'' + y = 0\,, y(0) = 0, y'(\pi) = 1$.

   b. Define Fourier series and Fourier coefficient of function.

6. **a.** Define odd and even function. Is the function cosec2x is even?

   b. Define half range sine series and cosine series.

Group B [13x4 = 52]

7. Define linear differential equation of first order. Solve IVP $ty' + 2ty = te^{-2t}\,,\ y(1) = 0$.
8. Define direction field, equilibrium solution. Consider the differential equation $\frac{dy}{dx} = -y + 2$, sketch the direction field and find the equilibrium solution.
9. Solve the IVP $y' = \frac{3x^3 + 4x + 2}{2(y-1)}\,, y(0) = 1$ and determine the interval.
10. Solve the IVP $y' = 2t(1 + y)\,,\ Y(0) = 0$ by method of successive approximation .
11. Find the solution of $y''' - y'' + y' - y = 0\,, y'(0) = -1\,,\ y''(0) = -2$. How does the solution when $t \to \infty$?

12. Solve the second order linear differential equation;

$y'' + p(x)y' + q(x)y = g(x)$ where $p, q, and\ g$ are continuous functions by the method of variation parameters.

13. State and prove the Abel's theorem.

14. Using the method of undetermined coefficient ,find the solution of the initial value problem :$y'' + 4y = t^2 + 3e^t,\ y(0) = 0\ , y'(0) = 2.$

OR

A body at the temperature of $50°$ F is placed outdoors where the temperature is $100°$ F. If after 5 minute the temperature of body is $60°$ F , find (a) how long it will take the body to reach a temperature of $75°$ F and (b) the temperature of body after 20 second.

15. Define linear independent and linear dependent set of solution of higher differential equation. Determine whether the given function are linear independent or dependent: $y_1(t) = 1\ , y_2(t) = 2 + t\ , y_3(T) = 3 - t^2\ , y_4(t) = 4t + t^2.$

16. Use the method of variation parameter $y''' - 2y'' - y' + 2y = e^{4t}$.

17. Define Bernoulli differential equation. Solve the equation: $t^2y' + 2ty - y^3 = 0\ , t > 0.$

18. Define Fourier Series and periodicity of sine and cosine function. Prove that $\int_{-L}^{L} cos\frac{m\pi x}{L} cos\frac{n\pi x}{L} dx = 0\ if\ m \neq n.$

19. Show that $f_1 = x^2 and\ f_2 = x^3$ are orthogonal function. prove that the sum and product of two even function are even. Also show that if f is even function, then $\int_{-L}^{L} f(x)dx = 2\int_{0}^{L} f(x)dx.$

OR

Prove that $\int_{-L}^{L} cos\frac{m\pi x}{L} cos\frac{n\pi x}{L} dx = \begin{cases} 0\ , m \neq n \\ L\ , m = n \end{cases}.$

Group C [4x6 = 24]

20. The field mouse population satisfied the differential equation:

$$\frac{dp}{dt} = 0.5p - 450$$

a. Find the time at which the population becomes extinct if $p(0) = 850$

b. Find the time of extinction if $p(0) = p_0\ where\ 0 < p_0 < 900.$

c. Find the initial population p_0 if the population it to becomes extinct in 1 year.

21. State the necessary and sufficient condition of exact differential equation. Find the integrating factor of differentia equation and solve; $(3xy + y^2) + (x^2 + xy)y' = 0.$

22. Define fundamental solution set of higher order differential equation. Define linearly independent and dependent set of solution. Determine whether the given functions $y_1(t) = 1, y_2(t) = 2 + t\ , y_3(t) = 3 - t^2\ , y_4(t) = 4t + t^2$ are linearly independent or dependent.

OR

Find the particular solution of $y''' - 4y' = t + 3cost + e^{-2t}$ by method of undetermined coefficient. Using method of variation of parameters solve $y''' - 2y'' - y' + 2y = e^{4t}.$

23. a. Find the general solution of $y^4 - y''' - 7y'' - y' + 6y = 0.$

b. Define homogeneous differential equation of first order. Solve the differential equation

$$\frac{dy}{dx} = \frac{xy - 4x^2}{x^2 - xy}$$

The End