

Mid-West University
Examinations Management Office

End Semester Examination 2081

Bachelor level/ B. Sc. Env/ 1st Semester

Time: 3 hours

Subject: Mathematics of Environment System (ENV412)

Full Marks: 60

Pass Marks:30

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group-A

Very short answer questions (Attempt Any Ten)

[10x2 = 20]

1. How many 3-digit numbers that can be formed from the given digits: 1, 2, 3, 4 and 5 assuming that digits are not allowed to be repeated.
2. Find the domain of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x-1}{x^2-1}$
3. If $A = \begin{pmatrix} 1 & 2 \\ -2 & 5 \end{pmatrix}$ then find A^{-1}
4. Solve the equation using matrix inverse method: $5x - 3y = 8$, $9x + 7y = 126$
5. Write the condition for a function to be continuous.
6. Evaluate the integral $\int x \log x \, dx$
7. Find $\frac{dy}{dx}$ if $y = (3x^2 + 5)^3$
8. Evaluate the limit $\lim_{x \rightarrow 2} \frac{x^4-1}{x^2-4}$
9. Compute the magnitude of $5i - 4j + 2k$.
10. The side of a square sheet is increasing at a rate of 3 cm/min. Find the rate of increasing the area of the square sheet when the side length is 12 cm.
11. Integrate: $\int \frac{x+a}{a^2+x^2} \, dx$
12. Find the volume of ellipsoid formed by the revolution of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis.

Group B

Short answer questions (Attempt Any Four)

[4x5 = 20]

13. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has at least one girl.
14. Show that the area bounded by the curve $y^2 = 4ax$ & $x^2 = 4ay$ is $\frac{16a^2}{3}$ square unit.
15. Show that
$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$
16. A function $f(x)$ is defined as follows
$$f(x) = \begin{cases} \frac{2x^2-18}{x-3} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$$
Find the value of k so that $f(x)$ is continuous at $x = 3$.
17. Find the derivative of $y = \frac{1}{\sqrt{3x+2}}$ by using first principles.

Group C

Long answer questions (Attempt *Any Two*)

[2x10 = 20]

18. There are 4 air flights and 10 buses per day to travel from Surkhet to Kathmandu. In how many ways a person can travel from Surkhet to Kathmandu?
Define one to one and onto function. Determine whether the function $f: [-2, 2] \rightarrow \mathbf{R}$ defined by $f(x) = x^2$ is one to one or not.
What do you mean by continuity of a function? Determine whether the function defined by $f(x) = \frac{x^2-9}{x-3}$ is continuous at $x = 3$.
19. Define symmetric and skew-symmetric matrix with example. Under what condition two matrices are comfortable for multiplication? If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and $f(x) = x^2 - 5x + 7$ Show that $f(A)$ is a null matrix. Solve the equations using Carmer's rule:
 $x + y + z = 2, x + z = 7, 2x + y + z = 13$ [1+1+1+3+4]
20. Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$. Find the local maxima and minima of the function $f(x) = x^3 - 12x + 18$. Also evaluate the integral $\int \frac{2x+5}{x^2+5x+6} dx$. And the area bounded by the X-axis, and the following curve and the ordinate: $x^2 = y, x = a, x = b$. [2+ 3+1+4]

The End