Mid-West University

Examinations Management Office

Final Examinations-2079

Bachelor level/ B.Sc /3rd Semester — Full Marks: 60

Time: 3 hours — Pass Marks: 30

Subject: Calculus III (MTH433/333)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Long questions **[4x6=24]**

1) Define tangential and normal components of the acceleration vector. If $\vec{r}(t) = e^t\vec{i} + \sqrt{2}\,t\,\vec{j} + e^t\,\vec{k}$, find tangential and normal components of the acceleration vector.
2) Find the local maxima, local minima and saddle point of function $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$.
3) Find the mass and center of mass of the lamina, D is bounded by $y = e^x, y = 0, x = 0$ *and* $x = 1$ and density function $\rho(x,y) = y$.

   OR

   Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.
4) State Greens' theorem. Use Greens theorem evaluate the line integral $\int_C x^2y^2dx + 4xy^3dy$, where C is the triangle with vertices (0,0), (1,3) and (0,3).

Group B

Short questions **[6x4=24]**

5) Show that the curvature of the curve given by the vector function $\vec{r}(t)$ is $\kappa(t) = \frac{\left|\vec{r'}(t)\times\vec{r''}(t)\right|}{\left|\vec{r'}(t)\right|^3}$. If $\vec{r}(t) = \langle e^t cost, e^t sint, t\rangle$, find curvature at point (1,0,0).
6) If $z = \ln(e^x + e^y)$, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (\frac{\partial^2 z}{\partial x \partial y})^2$.

   OR

   If $x^2 + y^2 + z^2 = 3xyz$, find $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$.
7) Evaluate the double integral $\iint_D (x+y)dA$, D is bounded by $y = \sqrt{x}$ and $y = x^2$.
8) Evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the rectangle that lies inside the cylinder $x^2 + y^2 = 16$ and between the plane $z = -5$ *and* $z = 4$.
9) Evaluate the line integral $\int_C \vec{F}.d\vec{r}$, where C is vector function $\vec{r}(t) = t^2\vec{i} + t^3\vec{j} + t^2\vec{k}$; $0 \le t \le 1$, $\vec{F}(x,y,z) = (x+y)\vec{i} + (y-z)\vec{j} + z^2\vec{k}$.
10) Find curl and divergence of vector field $\vec{F}(x,y,z) = e^{xy} sinz\vec{j} + y\,tan^{-1}(\frac{x}{z})\,\vec{k}$.

Group C

Very short questions **[(3x2)2=12]**

11) a) Find vector equation and parametric equation of line segment joining P(1,-1,2) and Q(4,1,7).
    b) Find directional derivative $f(x,y) = x^2y^3 - y^4$ at (2,1) in the direction $\theta = \frac{\pi}{4}$.
12) a) Find equation of tangent plane to the surface $z = \sqrt{xy}$ at (1,1,1).
    b) Use reimann sum with $m = 4, n = 2$ to estimate the value of $\iint_R (y^2 - 2x^2)dA$, where $R = [-1,3] \times [0,2]$. Take the sample points to be upper half corners of the square.
13) a) Change the rectangular to spherical coordinate of $(1,1,\sqrt{2})$.
    b) Find gradient vector field of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$.

THE END