Mid-West University

Examinations Management Office

Final Examinations-2079

Bachelor level/ B.Sc CSIT / 2nd Semester — Full Marks: 60

Time: 3 hours — Pass Marks: 30

Subject: Mathematics II (MTH425)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Long Answer Questions (Any Three) **[3x8=24]**

1 a Find the general solution of the system whose augmented matrix is : $\begin{bmatrix} 1 & 6 & 2 & -5 & -2 & -4 \\ 0 & 0 & 2 & -8 & -1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{bmatrix}$ [4]

b Show that following vector are linearly independent. [4]

$$v_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 18 \\ -4 \end{bmatrix}.$$

[8]

2 Find the LU factorization of A = $\begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix}$

Use this LU factorization of A to solve AX = b where $b = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.

3 a Find the dimension of null space and column space of : $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. [4]

b Let the two basis of $\boldsymbol{R^2}$ given by $B = \{b_1, b_2\}$ *and* $C = \{c_1, c_2\}$ where $\boldsymbol{b_1} = \begin{bmatrix} \mathbf{7} \\ \mathbf{5} \end{bmatrix}$, $\boldsymbol{b_2} = \begin{bmatrix} \mathbf{-3} \\ \mathbf{-1} \end{bmatrix}$, $\boldsymbol{c_1} = \begin{bmatrix} \mathbf{1} \\ \mathbf{-5} \end{bmatrix}$, $\boldsymbol{c_2} = \begin{bmatrix} \mathbf{-2} \\ \mathbf{2} \end{bmatrix}$ then find the change of coordinate matrix from B to C. [4]

4 a Find QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. [4]

b Define subspace of vector space. Is the set H of all matrices of the form $\begin{bmatrix} 2a & b \\ 3a+b & 3b \end{bmatrix}$ a subspace of $M_{2\times 2}$? [4]

Group B

Short Answer Questions (Any Five) **[5x4=20]**

5 Show that y be span of $\{v_1, v_2, v_3\}$ if [4]

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} \quad y = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

6 Define linear transformation. Let $T: R^2 \to R^3$ be define by $T(s,t) = (2s+3t,\ -s+5t\ , 4s-3t)$. Show that T is linear. [1+3]

7 Determine the rank of the matrix = $\begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$. [4]

8 Find the determinant by row reduced to echelon form: $\begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 4 \\ 3 & -1 & -2 & 3 \end{vmatrix}$ [4]

9 Find the coordinate vector $[X]_B$ of X relative to the given basis $B = \{b_1, b_2, b_3\}$ where $b_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, b_2 = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, b_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$, $X = \begin{bmatrix} -8 \\ -9 \\ 6 \end{bmatrix}$. [4]

10 Daigonalize the matrix :A = $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ [4]

11 Find an orthogonal basis and orthonormal basis for $\boldsymbol{R^3}$ starting with these two vectors $u = (1,-1,-)$ and $v = (1,0,-1)$ [4]

Group C

Very Short Answer Questions **[4x(2+2)=16]**

12 **a** Define linear equation and system of linear equation with example. [1+1]

b Define REF and RREF with example. [1+1]

13 **a** Write a vector equation and matrix that is equivalent to the given system of equation: [1+1]

$$y + 5z = 0$$
$$4x + 6y - 3z = 0$$
$$-x + 3y - 8z = 0$$

b Write the any four properties of determinant. [2]

14 **a** Prove that every vector has unique additive identity. [2]

b Find the matrix A such that $W = ColA$ where [2]

$$W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in R \right\}.$$

15 **a** Define characteristics equation. Also state the Cayley-Hamilton theorem. [1+1]

b Show that $\{u_1, u_2, u_3\}$ is an orthogonal set. Where [2]

$$u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} \frac{-1}{2} \\ -2 \\ \frac{7}{2} \end{bmatrix}$$

THE END