Mid-West University

Examinations Management Office

Surkhet, Nepal

Final Examinations -2079

Bachelor level/ B.Sc /6th Semester — Full Marks: 100

Time: 3 hrs — Pass Marks : 50

Subject : Discrete Mathematics (Math461)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A **[6(2+2)=24]**

1. a) Define even number. Express the fact 81270 is a multiple of 3 or 7 is true.
   b) Let C = {n ∈ ℕ: n is a multiple of 6},
   D = { n ∈ ℕ: n is a multiple of 2 and 3}, prove that C = D.
2. a) Show that 29 is a prime number.
   b) Show that: $\sim p \vee q$ and $p \Rightarrow q$ are logically equivalent.
3. a)Define converse of statement. Write converse of If n is multiple of 3 then n is not multiple of 7.
   b) For any $a, b, c \in \mathbf{N}$, if $a < b$ then $a + b < b + c$.
4. a) Prove that function f has a inverse if it is bijective.
   b) Express the repeating decimals $0.56\overline{78}$ as fraction.
5. a) Show that if $p\ and\ p'$ are prime and $p \setminus p'$, then $p = p'$.
   b) Construct the multiplication for $\mathbb{Z}_5$.
6. a) Show that $\binom{n}{k} = \binom{n}{n-k}\ for\ 0 \le k \le n$.
   b) Show that if x and y are invertible in $\mathbf{Z}_m$then xy are invertible in $\mathbf{Z}_m$.

Group B **[13x4=52]**

7. Prove that there is no greatest prime number. For any natural number n,
   $n^2 + n$ is an even number.
8. Define tautology and contradiction. Construct the truth table of $\sim p \Rightarrow \sim\ (q \vee r)$.
9. Prove that if X is a subset of Y, and X is infinite then Y is infinite. The set P of primes is infinite.
10. For all integers $x, y, z; show\ that\ (i) x + 0 = x, (ii)\ x - (y - z) = (x - y) + z$.
11. Define bijection. Prove that if f and g are bijection and g f defined, then the inverse of g f is $f^{-1}\ g^{-1}$.
12. If $x \in N\ and\ n \ge 2$ and n is composite then there is prime such that $p \setminus n$ and $p^2 \le n$. If $x \setminus 0$ for all $x \in Z$ but $0 \setminus x$ only when $x = 0$.
13. Define cardinality of a set. Prove that the set of rational numbers is countable.
14. A positive integer $n \ge 2$ has a unique prime factotization apart from the order of the factor.
15. Show that between any two distinct rational numbers there is another rational number. Write nine rational numbers between $\frac{17}{20}$ and $\frac{4}{5}$.

OR

In the recursion for $\sqrt{2}$, x_n is a rational number $\frac{p_n}{q_n}$, show that $p_n{}^2 - 2\ q_n{}^2 = 1$ for all n ≥ 2.

16. Let $S_n$be set of permutations of $\{1, 2, \ldots, n\}$, then (i) if $\pi, \sigma \in S_n$ then $\pi\sigma \in S_n$.

(ii) if $i \in \mathrm{S_n}$ be identity function then for $\sigma \in \mathrm{S_n}$, $i\sigma = \sigma i = \sigma$.

17. Use identity$(1+x)^m(1+x)^n = (1+x)^{m+n}$, prove that $\binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \cdots + \binom{m}{r}\binom{n}{0} = \binom{m+n}{r}$ $for\ m \geq r\ and\ n \geq r.$
18. Suppose we are given $m \geq 2$ $and\ an\ integer\ x$. The remainder r when x is divided by m satisfies $x \equiv r\ (mod\ m),\ 0 \leq r \leq \mathrm{m} - 1.$

OR

Define partition of a set. Let $S(n,k)$ be partition of a n-set X into k parts, show that $S(n,2) = 2^{n-1} - 1$

19. Define invertible element of $\mathbb{Z}_m$. Write invertible elements of $\mathbb{Z}_6$. The element $r \in \mathbb{Z}_m$ is invertible if and only if r and m are coprime in $\mathbb{Z}$.

Group – C [4x6=24]

20. Find the least member and the greatest member of the set $X = \{n \in \mathbb{N}/n^2 + 2n \leq 60\}$, show that for every $n \in \mathbb{N}$, $\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5)$.
21. If n is a positive integer, $n \geq 2$. Then n has a unique prime factorization,apart from the order of the factor. If $\gcd(a,b) = d$ and $a = da', b = db'$ show that $\gcd(a', b') = 1$.

22. Define Euler's function with example. Show that for any positive integer n ,

$$\sum_{d\backslash n} \Phi(d) = n .$$

OR

Let $n \geq 2$ be integer whose prime factors is $n = p_1^{e_1} p_2^{e_2} \ldots.. p_r^{e_r}$. Then $\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \ldots.(1 - \frac{1}{p_r})$. Also determine $\varphi(45)$.

23. The element $\mathrm{r} \in \mathbb{Z}_\mathrm{m}$ is invertible iff r and m are coprime in $\mathbb{Z}$. Construct the multiplication table of $\mathbb{Z}_6$.

THE END