

Mid-West University  
Examinations Management Office  
Surkhet, Nepal  
Examinations -2079

Bachelor level/ B.Sc /1<sup>st</sup> Semester

Time: 3hrs

**Subject : Calculus I (MATH413/313)**

Full Marks : 60

Pass Marks: 30

*Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.*

**Group – A**

**Attempt all the questions :**

**[6x4=24]**

1. State and prove Rolle's Theorem. Verify that the function satisfies the Rolle's Theorem in given interval,  $f(x) = x^3 - x^2 - 6x + 2$ , in the interval  $[0,3]$ . [4+2]
2. Let  $f(x) = \sqrt{x}$ ,  $[0,4]$  [2+2+2]
  - i. Find  $f_{ave}$  on  $[0,4]$
  - ii. Find  $C$  such that  $f_{ave} = f(C)$
  - iii. Sketch the graph of  $f$  and a rectangular whose area is the same as the area under the graph of  $f(x)$ .
3. a) Solve the differential equation  $y^2 \frac{dy}{dx} = 2x$ , find  $y$  if  $y(0) = 2$ .  
b) Find the area of the surface by rotating the given arc about corresponding axis.  $y = x^3 + 1, 0 \leq x \leq 2$  about x-axis. [3+3]
4. Explain the difference between an absolute minimum and a local minimum. Find the absolute maximum and absolute minimum values of  $f(x)$  on the given interval.  
 $f(x) = 2x^3 - 3x^2 - 12x + 1, [-2,3]$ . [2+4]

OR

- a) Evaluate the integral:  $\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} dx$ . [3+3]
- b) Find  $\int_0^2 \left( 2x^2 - 6x + \frac{3}{x^2+1} \right) dx$  and interpret the results in term of area.

**Group – B**

**Attempt all the questions :**

**[6x4=24]**

5. As dry air moves upward, it expands and cools, if the ground temperature is  $30^\circ$  and the temperature  $T$  (in  $^\circ\text{C}$ ) as a function of the height  $h$  (in km), assuming that linear model is appropriate
  - i) Draw the graph of the function.
  - ii) What is the temperature at a height of 2.5 km?
6. Define the arc length of the function. Find the length of the arc of the curve from point  $P$  to point  $Q$ ,  $x^2 = (y - 4)^3, P(1,5), Q(8,8)$ .

OR

7. Find the solution of the initial value problem,  $x^2 y' + xy = 1, x > 0, y(1) = 2$
8. Evaluate the integrate: i)  $\int \frac{4x^2 - 3x - 4}{x^3 + x^2 - 2x} dx$  ii)  $\int_0^{\frac{\pi}{2}} \cos^3 x \sin(2x) dx$ .
9. An airplane flying horizontally at an altitude of  $y = 3\text{km}$  and at a speed of 480km/hr passes directly above an observer on the ground. How fast is the distance 'D' from the observer to the airplane increasing 30sec later?
10. Find the equation of the tangent line to the curve at the given point  
 $x^2 + xy + y^2 = 3$  at  $(1,1)$ .
11. Define the work. When a particle is located  $x$  feet from the origin, a force of  $x^2 = 3x$  pounds acts on it. How much work is done in moving it from  $x = 2$  to  $x = 5$ ?

**Group – C**

**Attempt any six questions:**

**[6x2=12]**

12. a. Evaluate the limit:  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$ . [2]  
b. The function  $f(y) = \begin{cases} e^y & \text{if } y \leq 1 \\ my + b & \text{if } y > 1 \end{cases}$  is continuous and differentiate at  $x = 1$ , find the value of the constant  $m$  and  $b$ . [2]
13. a. Define the horizontal asymptotes. Find the horizontal asymptotes of the function  $f(x) = \frac{x-3}{\sqrt{x^2-9}}$ . [1+1]  
b. A curve is defined by the parameter equations  $x = 3(t^2 - 3), y = t^3 - 3t$ . Calculate  $\frac{dy}{dx}$  in term of  $t$ . [2]
14. a. Evaluate the integral:  $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x) dx$ . [2]  
b. Find the average value the function  $f(x) = 1 + x^2$  on the interval  $[-1,2]$ . [2]

**THE END**