Mid-Western University

Examinations Management Office

Final Examinations -2079

Bachelor level/ B.Sc /6th Semester — Full Marks : 100

Time: 3 hrs — Pass Marks : 50

Subject : Modern Algebra I (MATH463)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group- A

Attempt all the questions. **[6 x (2+2) = 24]**

1. a) Let I be the set of integers. Let us define a relation R on I such that aRb holds if $a - b$ is division by 5. Show that the relation is transitive.
   b. The inverse of the product of the of two elements of a group G is the product of the inverse taken in the inverse order.
2. a) $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ then find $f \circ f$
   b. If a, b, c are any element of a group $(G,*)$ then $a * b = a * c \Longrightarrow b = c \& b * a = c * a \Longrightarrow b = c$
3. a) Define a subgroup and give an example.
   b) Prove that arbitrary intersection of subgroups of a group is a sub-group
4. a) Prove that every cyclic group is abeilan.
   b) Define integral domain and give an example of it.
5. a) Show that every field has an integral domain.
   b) Prove that the skew field has no zero divisors
6. a) Define field with suitable example.
   b) What is extension and degree of extension of field?

Group -B (13 x 4 = 52)

7. Define congruence modulo m. Show that "congruence modulo "is an equivalence relation in the set of integers.
8. Let G be a group, if every element of G is its inverse, prove that G is an abelian group and also in a group G if $x * x = x$, then $x = e$.
9. Let $(\boldsymbol{G},*)$is a group then a finite subset H of G is a subgroup of G iff i) $H \neq \emptyset$ ii) $a * b \in H, \forall a, b \in H$
10. If H,K are two subgroups of a group G then HK is a subgroup of G iff HK = KH.
11. Define additive modulo n. Solve for x if 2x + 1 = 3 in Z_5, where Z_5 denotes the addition and multiplication modulo 5 in the set Z.

OR

State and prove Cayley's Theorem.

12. Define cyclic group with a suitable example. Let G be a cyclic group and H is a subgroup of G, prove that H is also cyclic.
13. Give the meaning of homomorphism of groups. If $\emptyset$ is a homomorphism of G into G' then (i) $\emptyset(e) = e' \& \emptyset(x^{-1}) = [\emptyset(x)]^{-1}, \forall x \in G$
14. Define kernel of homomorphism. A homomorphism $\emptyset$: G → G 'with kernel K_ϕ is an isomorphism of G on to G' iff $K_\phi = \{e\}$.
15. Let R be a ring and S and T are two subrings of R, then prove that S ∩ T is also a subring.
16. Prove that a finite commutative ring without zero deviser is a field .
17. Define maximal ideal of a ring. An ideal S of a ring of integers I is maximal, if S is generated by some prime integer
18. Define skew field. Prove that a skew field has no zero divisors

OR

Show that a finite integral domain is a field.

19. Define integral domain. Prove that every field is an integral domain

Group- C **[4 x 6 = 24]**

20. Define generator of a cyclic group. Find the generator of the group G = {1, -1, i, - i}, where i is an imaginary unit. Prove that the order of a subgroup of a finite group is divisor of the order of the group.

21. If G is a group and H is a subgroup of G. Then

    i) if $(a*H) \cap (b*H) \neq \phi$, then $(a*H) = (b*H)$

    ii) if $(a*H) \neq (b*H)$, then $(a*H) \cap (b*H) = \phi$

OR

Every group is isomorphism to a group of permutation

22. State and prove Lagrange's Theorem.
23. An ideal S of ring of integers I is maximal iff S is generated by some prime number.

THE END