Mid-West University

# Examinations Management Office

Birendranagar, Surkhet

**End Semester (Alternative/Physical) Examinations -2078**

Bachelor level/ B.Sc /6th Semester — Full Marks : 60

Time: 3hrs — Pass Marks :30

**Subject : Discrete Mathematics (MATH461)**

*Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.*

**Attempt the following Questions** **[6x10=60]**

1. a) Define prime number. Write the counter-example for the statement $6n + 1$ is a prime. Prove that there is no greatest prime number. For any natural number $n$, $n^2 + n$ is an even number. **[5]**

   b) Show that $\neg (p) \lor q$ and $p \Rightarrow q$ are logically equivalent. Prove that if X is a subset of Y, and X is infinite then Y is infinite. Also prove that the set P of primes is infinite. **[5]**

2. a) Construct the counter-example for the statement $A \cap (B \cup C) = (A \cap B) \cup C$. Define Conjuction and disjunction. Construct the truth table of $p \Rightarrow \sim (q \lor r)$. **[5]**

   b) Define converse of statement. Write converse of "If n is multiple of 3 then n is not multiple of 7". Define injection and surjection functions. Suppose $f, g$ *and* $h$ are functions such that composite $h(fg)$ is defined. Show that $(hg)f = h(gf)$. **[5]**

3. a) For any $a, b, c \in \mathbf{N}$, if $a < b$ then $a + b < b + c$. If $x \in N$ *and* $n \geq 2$and n is composite then there is prime such that $p \backslash n$ and $p^2 \leq n$. If $x \backslash 0$ for all $x \in Z$ but $0 \backslash x$ only when $x = 0$. **[5]**

   b) Prove that function $f$ has a inverse if it is bijective. Show that between any two distinct rational numbers there is another natural number. Write nine rational numbers between $\frac{17}{20}$ and $\frac{4}{5}$. **[5]**

   **OR**

   Let $S_n$be set of permutations of $\{1, 2, \dots, n\}$, then (i) if $\pi, \sigma \in S_n$ then $\pi\sigma \in S_n$.(ii) if $i \in S_n$ be identity function then for $\sigma \in S_n$ $i\sigma = \sigma i = \sigma$.Use identity$(1 + x)^m (1 + x)^n = (1 + x)^{m+n}$, prove that
   $\binom{m}{0}\binom{n}{r} + \binom{m}{1}\binom{n}{r-1} + \cdots + \binom{m}{r}\binom{n}{0} = \binom{m+n}{r}$ *for* $m \geq r$ *and* $n \geq r$. **[10]**

4. Let $x, y \in \boldsymbol{Z}, x \neq 0, y \neq 0$ *and* $x \backslash y,\ y \backslash x;$ *then* $x = y$ *or* $x = -y$. If n is a positive integer, $n \geq 2$. Then n has a unique prime factorization,apart from the order of the factor. If $\gcd(a, b) = d$ *and* $a = da', b = db'$ show that $\gcd(a', b') = 1$. **[10]**

5. a) Let $n \geq 2$ be integer whose prime factors is $n = p_1^{e_1} p_2^{e_2} \dots . . p_r^{e_r}$. Then $\varphi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \dots . (1 - \frac{1}{p_r})$. Also determine $\varphi(45)$. **[5]**

   b) Define principle of induction. Use the strong form of the principle of induction to show that if $x_1 = 3, x_2 = 5, x_{n+1} = 3x_n - 2x_{n-1}, \forall n \geq 2,$ *then* $x_n = 2^n + 1, \forall n \in \mathbf{N}$. **[5]**

   **OR**

   Define invertible element of $\mathbb{Z}_m$. Write invertible elements of $\mathbb{Z}_6$. The element $r \in \mathbb{Z}_m$ is invertible if and only if $r$ *and* $m$ are coprime in $\mathbb{Z}$. Given any positive integer $n, n_1, n_2, \dots, n_k$ satisfying $n_1 + n_2 + \cdots + n_k =\ n,$ *then* $\binom{N}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1! n_2! \dots n_k!}$. Show that if $a + b + c = p,$ *then* $\binom{p}{a, b, c} = \binom{p-1}{a-1, b, c} + \binom{p-1}{a, b-1, c} + \binom{p-1}{a, b, c-1}$.

6. Define cardinality of a set. Prove that the set of rational numbers is countable. Define integral modulo $m$.Prove that the operation addition $\oplus$ and multiplication $\otimes$ defined on integral modulo m of $\mathbb{Z}_m$ satisfies. **[10]**

   i) $a \oplus b = b \oplus a$, $a \otimes b = b \otimes a$.

   ii) $a \otimes (b \otimes c) = (a \otimes b) \otimes c$.

   iii) $a \otimes 1 = a$.

**THE END**