Mid-West University

Examinations Management Office

Birendranagar, Surkhet

End Semester (Alternative/Physical) Examinations -2078

Bachelor level/ B.Sc /4th Semester — Full Marks : 60

Time: 3hrs — Pass Marks : 30

Subject : Linear Algebra I (MATH 343)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions. **[6 x 10 = 60]**

1. a) Is the system – x + y = 1and 2x + y = 4 consistent? Solve the system of equations $2x_1 + 3x_2 - x_3 = -5$, $4x_1 - x_2 + 2x_3 = 24$, $3x_1 - 3x_3 + x_2 = -8$ by row – echelon form method.

   b) Is the vector w = $[7, 6, -5]^T$ in kernel of this matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$? Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ -2 & -1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$

2. a) Find the value of x when Det $\begin{bmatrix} 3 & 5 & 0 \\ 2 & 7 & 0 \\ x & 1 & x \end{bmatrix} = 10$. Find the left inverse of this matrix $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

   b) Consider $t \begin{bmatrix} 12 & 12 \\ 12 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$. Use the determinantal criterion for non invertibility to find all the values of t for which this matrix is non – invertable.

3. a) Find the centre of mass of the system $x_1 = (1, 3, 2)$, $x_2 = (-2, 1, 0)$, $x_3 = (-3, 2, 2)$, if the weights are 3, 7, 5 are respectively. Define invertible matrix. Here are two matrices

   $A = \begin{bmatrix} 3 & 7 & 5 \\ 5 & 11 & 8 \\ 3 & 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 9 & -6 & 1 \\ -13 & 9 & -2 \end{bmatrix}$ If one of these the inverse of the other?

   b) If A is invertible, then the solution of the system of equation $A\vec{x} = \vec{b}$ is given by the formula $x_j = \frac{\text{Det}[A_j(\vec{b})]}{\text{Det}(A)}$, $(1 \le j \le n)$ where $A_j(\vec{b})$ is matrix A which its j^{th} replaced by $\vec{b}$.

   OR

   a) If $\vec{u}$ is a vector such that $A\vec{u} = \vec{b}$, then every solution of the equation $A\vec{x} = \vec{b}$ is of the form $\vec{x} = \vec{u} + \vec{z}$ for some vector $\vec{z} \in$ ker (A). Let L_1 and L_2 be any two lines in $\mathcal{R}^n$ described by Let $L_1 = \{ u + tv : t \in \mathcal{R} \}$ and $L_2 = \{ w + sz : s \in \mathcal{R} \}$. Prove that these two lines are same iff u – w and v are multiple of z.

   b) A plane in $\mathbb{R}^3$, contains the three points: $\vec{u} = (15, 5, 2)$, $\vec{v} = (6, 2, 1)$ and $\vec{w} = (10, 3, 2)$. What is the standard form of this plane?

4. Does the line intersect? Where $\vec{p} = (0, 7) + s(14, -7)$ and $\vec{q} = (0, 3) + t(3, 6)$. Find the value of β so that $(\beta, 3, -5)$ is a linear combination of this set of vectors {(1, 3, - 1), (-5, -5, 2)}.

5. In $\mathbb{R}^2$, if a triangle has vertices **o, u, and v**, then the area of a triangle is one half of the absolute value of 2 x 2 determinant having rows (or columns) **u and v**. Then Area [Δ(**o, u, v**)] = $\frac{1}{2}\left|Det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}\right|$.Define linear transformation. Let u = (1, 1, 3), v = (3, 2, - 2), L (u) = (4, 1, 1, 1), L (v) = (- 5, 1, -3, 3). Suppose L: $\mathcal{R}^3 \to \mathcal{R}^4$ is linear. If w = (5, 4, 4) and y = (2, 1, 7). Find L (w) and L (y).

OR

Solve the system of equations x + 3y + 2z = 1, 2x + y + z = 0, 4x – y + 3z = 0 by Cramer's rule. List the methods to solve the system of linear equations. Solve the given system of linear equation in any method do you like and verify your answer.

$$x_1 + 3x_2 + x_3 = 6$$
$$2x_1 + 6x_2 + 3x_3 = 16$$
$$3x_1 + 9x_2 + 4x_3 = 22$$

6. The determinant of a matrix and the determinant of its transpose are equal. That is, Det (A) = Det (A^T) . Define singular and non-singular matrix. For two square matrices A and B of same size, prove that Det (AB) = Det (A) Det (B).

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