Mid-West University

Examinations Management Office

Surkhet,Nepal

End Semester Examinations -2078

Bachelor level/ B.Sc / 3rd Semester | Full Marks : 100

Time: 3 hrs | Pass Marks : 50

Subject : Calculus-III (MATH 333)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

Attempt all the questions **[6(2+2)=24]**

1. a. Find the limit $\lim_{t\to\infty}\left(e^{-3z}\vec{i}+\frac{t^2}{sin^2t}\vec{j}+cos2t\vec{k}\right)$.
   b. Find the length of the curve $\boldsymbol{r}(t)=\boldsymbol{i}+t^2\mathbf{j}+t^3\boldsymbol{k},\quad 0\le t\le 1.$
2. a.Find the unit tangent vector of $\vec{r}(t)=(1+t^3)\vec{\imath}+te^{-t}\vec{\jmath}+sin2t\,\vec{k}$, at the point where $t=0$.
   b. Find the domain of the function $\frac{\sqrt{y-x^2}}{1-x^2}$.
3. a.Show that the function $u(x,y)=e^x siny$ satisfy the Laplace's equation.
   b. Calculate f_{xxyz} if $f(x,y\,z)=\sin(3x+yz)$.
4. a. Evaluate the iterated integral $\int_0^2\int_y^{2y}xy\,dx\,dy$.
   b. Change from rectangular coordinate to spherical form of $(-\sqrt{3},-3,-2)$.
5. a. Find the Jacobian of transformation of $x=5u-v, y=u+3v$.
   b. Find the Curl of $\mathbf{F}(x,y)=xe^y\mathbf{j}+e^z y\ \mathbf{k}$.
6. a. If $\vec{F}(x,y,z)=xz\vec{\imath}+xy\vec{\jmath}-y^2\vec{k}, find\ div\vec{F}$.
   b. Is the vector field $\vec{F}(x,y,z)=2xy\,\vec{\imath}+(x^2+2yz)\vec{\jmath}+y^2\vec{k}$ conservative? Justify it.

Group B

Attempt all the questions **[4×13=52]**

7. If $\vec{u}$ and $\vec{v}$ are differentiable vector function then prove that $\frac{d[\vec{u}(t)\times\vec{v}(t)]}{dt}=\vec{u}'(t)\times\vec{v}(t)+\vec{u}(t)\times\vec{v}'(t)$.
8. A particle moves with position function $\vec{r}(t)=\langle t,t^2,3t\rangle$. Find the tangencial and normal components of acceleration.
9. Find the equation of tangent plane to the given surface $z=y\cos(x-y)$at the given point (2, 2, 2).

   OR

   Find first partial derivative of the function $f(x,y)=\frac{ax+by}{cx+dy}$.
10. The Kinectic energy of body with mass *m* and velocity *v* is $K=\frac{1}{2}mv^2$. Show that $\frac{\delta K}{\delta m}.\frac{\delta^2 K}{\delta v^2}=K$
11. If $z=f(x,y)$ has continuous second order partial derivatives and $x=r^2+s^2$and $y=2rs$, find $\frac{\partial z}{\partial r},\frac{\partial^2 z}{\partial r^2}$.
12. Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$and above the region D in the xy-plane bounded by the line $y=2x$ and parabola $y=x^2$.

13. Evaluate the line of integral $\int_C (x^2y^3 - \sqrt{x})dy$,where C is the arc of the curve $y = \sqrt{x}$ form (1,1) to (4 , 2).
14. Evaluate the triple integral $\iiint_B xyz^2\, dV$, where B is the rectangular box given by $B = \{(x, y, z){:}\ 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$.
15. Let X and Y be random variables defined by

$$f(X, Y) = \begin{cases} 0.1e^{-(0.5x+0.29)} & if\ X \ge 0, Y \ge 0 \\ 0\ otherwise \end{cases}.$$

Verify that f is joint density function. Find $P(Y \ge 1) and\ P(X \le 2, Y \le 4)$.
16. The integral $\int_C \vec{F}.d\vec{r}$ is independent of path in D if and only if $\int_C \vec{F}.d\vec{r}$=0, for every closed path C in D.

OR

If f is a function of three variables that has continuous second order partial derivatives then $curl(\nabla f) = 0$.
17. Show that $F(x, y, z) = xyz^2\vec{\imath} + 2x^2yz^2\vec{\jmath} + 3x^2y^2z\vec{k}$ is a conservative vector field.
18. If $\vec{F} = P\vec{\imath} + Q\vec{\jmath} + R\vec{k}$ is a vector field on R^3 and P,Q, and R have continuous second order partial derivatives, then $div\ curl\vec{F} = 0$.
19. Verify the Divergence Theorem for the vector field **F**(x, y, z)= $3x\,\boldsymbol{i} + xy\,\mathbf{j} + 2xz\,\boldsymbol{k}$, on the region E bounded by the planes $x = 0, x = 1 , y = 0, y = 1, z = 0$ & $z = 1$

Group C

Attempt all the questions **[6×4=24]**

20. Prove that for plane curve, curvature is $\kappa(t) = \frac{|f''(x)|}{[1+(f'(x)^2]^{\frac{3}{2}}}$.

Find the curvature of the $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.
21. If $u = f(x, y)$, where $x = e^s cost\ and\ y = e^s sint$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}[\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}].$$

OR

If $z = f(x, y)$ is a differential function of x and y where x and y are function of t ,then z is a function of t and $\frac{dz}{dt} = \frac{\delta f}{\delta x} . \frac{dx}{dt} + \frac{\delta f}{\delta y} . \frac{dy}{dt}$
22. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.
23. State and prove Green's theorem.

THE END