Mid-West University

Examinations Management Office

Birendranagar, Surkhet

End Semester (Alternative/Physical) Examinations -2078

Bachelor level/ B.Sc /6^{th} Semester — Full Marks : 60
Time: 3hrs — Pass Marks : 30
Subject : Modern Algebra I (MATH463)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions. **[6x10) = 60]**

1. a) Define group with a suitable example. Let G be a group, if every element of G is its inverse, then prove that G is an abelian group. Also in a group G if $x * x = x$, then $x = e$.
   b) Define cyclic group with a suitable example. Let G be a cyclic group and H is a subgroup of G, prove that H is also cyclic.
2. a) Prove that every subgroup of an abelian group is a normal subgroup. State and prove Cayley's Theorem.
   b) If G is a group and H is a subgroup of G. Then
      i) if $(a *H) \cap (b *H) \neq \phi$, then $(a *H) = (b *H)$
      ii) if $(a *H) \neq (b *H)$, then $(a *H) \cap (b *H) = \phi$
3. a) Solve for x, $3x = 1$ in Z_4, where Z_4 denotes the addition modulo 4. Define additive modulo n. Solve for x if $2x + 1 = 3$ in Z_5, where Z_5denotes the addition and multiplication modulo 5 in the set Z.
   b) Define homomorphism. A homomorphism $\emptyset : G \to G'$ with kernel K_ϕ is an isomorphism of G on to G' iff $K_\phi = \{e\}$.

OR

Define subgroup of a group with a suitable example. Define a subgroup of a group with a suitable example. Let $(G, *)$ be a group and, then the finite subset H of G is a subgroup of G iff i) $H \neq \phi$ ii) $a * H \in H, \ \forall a, b \in H$.

4. Prove that the intersection of two subrings of a ring R is also a subring. Define subgroup of a group with suitable example. State and prove Lagrange's Theorem.
5. Prove that a skew field has no zero divisor. If S is an ideal of ring R. Then the set $\frac{R}{S} = \{S + a : a \in R\}$ of all residue classes of S in R form a ring for two composition in $\frac{R}{S}$ defined as i) $(a + S) + (b + S) = (a + b) + S$, and ii) $(a + S)(b + S) = ab + S$.

OR

Define skew field. Prove that a skew field has no zero divisors. Also prove that every quotient ring is a isomorphic image of the ring R.

6. a) Define subring of a ring. Let R be a ring and S and T are two subrings of R, then prove that $S \cap T$ is also a subring.
   b) An ideal S of ring of integers I is maximal iff S is generated by some prime number.

THE END