Mid-West University

Examinations Management Office

End Semester Examinations 2081

Bachelor level/ B.E. Computer/ 1st Semester — Full Marks: 50
Time: 3 hours — Pass Marks: 25
Subject: Engineering Mathematics-I (SH411/SH501)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a) **i.** If $y = e^{ax} \sin bx$, then prove that $y_{n+1} = 2ay_n - (a^2 + b^2)y_{n-1}$. **(2+3)**

ii. Evaluate: $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

b) State and prove the Rolle's theorem. Also write the geometrical interpretation of Rolle's theorem. **(4+1)**

2. a) **i.** Show that the radius of curvature at any point (x, y) of the curve $ay^2 = x^3$. **(3+2)**

ii. Find the asymptotes of the curve: $r\cos\theta = a\sin\theta$

b) **i.** Define Bernoulli's Equation and solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$. **(3+3)**

ii. Integrate: $\int \frac{1}{5+4\cos x}\,dx$.

3. a) Define the Beta and Gamma function. Use the Beta and Gamma function, Show that $\int_0^{\frac{\pi}{6}} \cos^2 6\theta \sin^4 3\theta d\theta = \frac{7\pi}{192}$. **(1+3)**

b) **i.** Find the area of the loop of the curve $y^2 = x^2(x + a)$. **(2+3)**

ii. Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

4. a) **i.** Find the equation of the tangent to the ellipse $3x^2 + 2y^2 = 1$ perpendicular to the line x+3y=1. **(3+2)**

ii. Find the center, length of axes, eccentricity and directrix of the ellipse $2x^2 + 3y^2 - 4x + 5y + 4 = 0$.

b) **i.** What does the equation $3x^2 + 2xy + 3y^2 = 2$ become when the axes are turned through an angle 45^0 to the original axes? **(2+3)**

ii. Find the centre, length of the axes and eccentricity of the conic $9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$.

5. a) **i.** Solve the differential equation: $\frac{dy}{dx} + y\cot x = 2\cos x$. **(2+3)**

ii. Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}\sin x$.

b) Define Homogenous linear differential equation and Solve: $x^2\frac{d^2x}{dx^2} - 2x\frac{dy}{dx} + 2y = \frac{1}{x}$. **(1+4)**

The End