Mid-West University

Examinations Management Office

Final Examinations -2081

Level: Bachelor level/B.Sc. CSIT/ Semester II

Time: 3hrs.

Subject: Mathematics II (MTH 425)

F. M: 60

P. M: 30

Candidates are required to give their answer in their own words as far aspracticable. The figures in the margin indicate full marks.

Group A

Very short Answer Questions [4x(2+2)=16]

1. a) Define consistent and inconsistent system of equations. Is the given system $x+y=1$ and $x+y=4$ consistent? [2]

   b) Give suitable example of REF and RREF. [2]

2. a) Let $A=\begin{bmatrix}2 & 5\\ 3 & 1\end{bmatrix}, u=\begin{bmatrix}4\\ -1\end{bmatrix}, v=\begin{bmatrix}-3\\ 5\end{bmatrix}$, verify that $A(u+v)=Au+Av$. [2]

   b. Define null space. If $X=\begin{bmatrix}-2\\ 1\end{bmatrix}$ and $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$, Is $X\in$ Null A. [2]

3. a) Define vector space with suitable example. [2]

   b) Let W be the set of all vectors of the form $\begin{bmatrix}5b+2c\\ b\\ c\end{bmatrix}$, where b and c are arbitrary constant. Find the vector u and v such that $W=span\{u,v\}$ [2]

4. a) Define eigenvalue and eigenvector. Find the eigenvalue of matrix $A=\begin{bmatrix}2 & 4 & 7\\ 0 & 3 & -5\\ 0 & 0 & 1\end{bmatrix}$ [1+1]

   b) Define inner product of vectors. Is $u.v=v.u$? Justify your answer. [2]

Group B.

Short Answer Questions (Attempt any five) [5 x 4 =20]

5. What do you mean by linear combination of vectors? Let $u=\begin{bmatrix}2\\ -3\\ 2\end{bmatrix}$ and $A=\begin{bmatrix}5 & 8 & 7\\ 0 & 1 & -1\\ 1 & 3 & 0\end{bmatrix}$ Is u in the subset of spanned by the column of A? Justify your answer. [1+3]

6. Define linear transformation. Is the transformation $T: R^2\rightarrow R^3$ defined by $T(s,t)=(s+t,-s+2t+3,4s-3t)$ linear? Give the reason. [1+3]

7. Let $A=\begin{bmatrix}A_{11} & a_{12}\\ 0 & A_{22}\end{bmatrix}$ where A_{11} is $p\times p$, A_{22} is $q\times q$ and A is invertiable. Find a formula for A^{-1}. [4]

8. Find the determinant by row reduced to echelon form:

$$\begin{bmatrix}1 & -1 & -3 & 0\\ 0 & 1 & 5 & 4\\ -1 & 2 & 8 & 4\\ 3 & -1 & -2 & 3\end{bmatrix}$$ [4]

9. Define coordinate vector. Find the coordinate vector $[X]_B$ of X relative to the given basis $B=\{b_1,b_2,b_3\}$ where $b_1=\begin{bmatrix}1\\ -1\\ -3\end{bmatrix}$, $b_2=\begin{bmatrix}-3\\ 4\\ 9\end{bmatrix}, b_3=\begin{bmatrix}2\\ -2\\ 4\end{bmatrix}$ & $X=\begin{bmatrix}8\\ -9\\ 6\end{bmatrix}$ [1+3]

10. Define orthogonal set of vectors. Let $\{u_1, u_2, \ldots u_p\}$ be an orthogonal basis for a vector space W of R^n. Show that for any y in W, the weights in the linear combination is $y = c_1u_1 + c_2u_2 + \ldots\ldots c_pu_p$ are given by: $c_j = \frac{y.u_j}{u_j.u_j}$ where j = 1,2,3,....p. [4]

Group C

Long Answer Question (Attempt any three) [3x 8 =24]

11. What are the types of elementary row operations? Define basic variables and free variables with example? Determine if the following homogeneous system has a nontrivial solution, then describe the solution set. [1+2+5]

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 2x_2 + 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

12. a) Is the matrix multiplication satisfy the commutative property? Justify your answer. [2]

b) Use the crammer's rule to solve the system:

$5\frac{1}{x} - 3y = 9$ and $2\frac{1}{x} + 5y = 16$. [2]

c) Diagonalize the matrix: $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ [4]

13. Define row space, null space and the column space. Find the orthogonal complement of row space of A.

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$ [1+1+1+5]

14. a) Find QR factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ [6]

b) Use Gram-Schmidit orthogonal process to produce an orthogonal basis for W, where $W = \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \\ -9 \\ 3 \end{bmatrix} \right\}$ [2]

The end