Mid-West University

Examinations Management Office

Final Examinations -2081

Level: Bachelor level/Science/ fourth Semester — F. M: 60

Time: 3hrs. — P. M: 30

Subject: Linear Algebra I (MTH343/443)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group-A [$4\times 6 = 24$]

1. What is the difference between REF and RREF? Find the general solution of the linear system: $\begin{bmatrix} 1 & -2 & -1 & 3 & 0 \\ -2 & 4 & 5 & -5 & 3 \\ 3 & -6 & -6 & 8 & 2 \end{bmatrix}$ [1+5]

OR

What are the types of elementary row operations? What do you mean by basic variables and free variables? Show that the system of equations

$$3x - y + 2z = 1,\quad x + 2y - z = 3,\quad 2x - 2y + 3z = 2$$

is a consistent and solve them reducing into row reduce echelon form. [1+1+4]

2. Define transpose of matrix with an example. Show that for any two matrices of same order A and B, $(A+B)^T = A^T + B^T$. Also verify that the matrix multiplication do not commute. [1+3+2]

3. Define linear transformation. Is the mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ define by $T(x,y,z) = (2x+z, 3x+y+4, 5x+y-z)$ linear? Justify your opinion. Let T be a linear transformation from $\mathbb{R}^n$ *to* $\mathbb{R}^m$. Then show that there is an $m\times n$ matrix such that $t(x) = Ax$ for all x *in* $\mathbb{R}^n$. **[1+2+3]**

4. Define vector space. Show that $\mathbb{R}^3$ is a vector space. Also prove that every vector has unique additive inverse. **[4+2]**

Group B **[6 x 4=24]**

5. Define the linear combination of a vector. Is the vector $w = (-1, 3, 7)$ a linear combination of the vectors $u = (4,2,7)$ and $v = (3,1,4)$? Justify your answer. **[1+3]**

6. Write the condition that the two lines described by $L_1 = \{u + tv : t \in R\}$ *and* $L_2 = \{w + sz : s \in R\}$ are

   i) same ii) parallel and distinct iii) intersect each other. If $u = (4,2,1), v = (-1,3,2), w = (1,11,7)$ *and* $z = (3,-9,-6)$. Determine whether the given lines are same or not. **[0.5+0.5+0.5+2.5]**

7. Define three polynomials as follows:

   $p_1(t) = t^3 + t, p_2(t) = t^2 + 1$ *and* $p_3(t) = 3t^3 - 2t^2 + 3t - 2$. Is the set $\{p_1, p_2, p_3\}$ linearly independent? Why? **[4]**

8. Let $L:\mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation for which $u = (1,1,3)$, $v = (3,2,-2), w = (5,4,4), L(u) = (4,1,1,1)$ *and* $L(v) = (-5,1,-3,3)$ then find $L(w)$. **[4]**

or

Prove that in R^2 counterclockwise rotating of every point by an angle $\emptyset$ is a linear transforming whose matrix $\begin{pmatrix} cos\emptyset & -sin\emptyset \\ sin\emptyset & cos\emptyset \end{pmatrix}$. **[4]**

9. Define invertible matrix. Find the left inverse of the matrix $\begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix}$

[1+3]

10. Mention any two properties of determinant. Use the crammer rules solve the following system of equations, $\begin{bmatrix} 3 & 2 & 7 \\ 1 & -4 & 1 \\ 4 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix}$. [1+3]

Group C [6 x 2=12]

11. Define consistent and inconsistent system of equations. Is the given system of equations $-x+y=1$ and $2x+y=4$ consistent. [1+1]

12. Find the center of mass of the system $x_1 = (1,3,2), x_2 = (-2,1,0),\ x_3 = (-3,2,2)$, if the weights are 3 ,7 and 5 kilos respectively. [2]

13. Define Null space. If $X = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Is $X \in Null\ A$. [1+1]

14. Define standard unit vector in $\mathbb{R}^3$. Express the vector x= (x_1, x_2, x_3) in term of standard unit vector. [1+1]

15. By using determinant, find the area of the triangle whose vertices are (4,7), (-2,11) and (12, -6). [2]

16. For what value of h in y in the plane spanned by v_1 *and* v_2, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$ *and* $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$? [2]

The end