Mid-West University

Examinations Management Office

End Semester Examinations 2081

Bachelor level/ B.Sc. / 6th Semester | Full Marks: 60
Time: 3 hours | Pass Marks: 30
Subject: Modern Algebra I (MATH463)

Candidates are required to give their answer in their own words as far as Practicable. The figures in the margin indicate full marks.

Group A [4x6=24]

1. Let G be a cyclic group generated by a,
   a) If G is finite order n then an element $a^k \in G$ is agenerator of G iff k and n are relatively prime. [2]
   b) If G is a finite order then show that a and a^{-1} are the only generator of G.[2]
2. Define factor group. Let H be the normal subgroup of group G and $\varphi : G \to \frac{G}{H}$ is a mapping define by $\varphi(a) = aH, \forall\, a \in G$, then show that φ is isomorphism and $ker\varphi = H$. [1+5]

OR

Define normal subgroup with example. If H , K are normal subgroups of a group G with $H \cap K = \{e\}$ then show that every element of H commute with every element of K. [1+1+4]

3. a) Define Abelian group. if g is abelian group then show that $(ab)^n = a^n b,^n \forall n \in \mathbb{Z}$ [1+2]
   b) Prove that the intersection of two subring of a ring is also ring. [3]
4. If S is an ideal of ring R then the set $\frac{R}{S} = \{S + a : a \in R\}$ of all residue classes of S in R form a ring for the two composition define
   a) $(a + S) + (b + S) = (a + b) + S$
   b) $(a + S)(b + S) = (ab) + S$. [3+3]

Group B [6x4=24]

5. Define isomorphism of group. Let $(\mathbb{Z}, +)$ and $(E\,, +)$ be two group under addition then show that the mapping $\varphi: \mathbb{Z} \to E$ define by $\varphi(n) = 2n$ is isomorphism . [1+3]
6. Define homomorphism. Let G and H are two subgroups and $\varphi : G \to H$ is homomorphism , then show that $ker\varphi$ is normal subgroup. [1+3]

OR

Prove that a group of order n is cyclic iff it has an element of order n.[4]

7. Define kernel of homomorphism. Let G and H be the two groups and $\varphi : G \to H$ is a homomorphism then show that $ker\varphi$ is a normal subgroup. [1+3]
8. Define Cayley Table. Let $G = \{1, -1, i, -i\}$ draw Cayley Table. Also define Klien four grpup with their properties. [1+1+2]
9. Define integral domain. Prove that every field in integral domain. [1+3]
10. Define commutative ring. Prove that the finite commutative ring without zero divisor is a field. [1+3]
11. Define subring of a ring. A non-empty subset S of a ring is a subring of R iff for $a\,, b \in S$ i) $a - b \in S$ ii) $ab \in S$. [1+3]

Group C [6 x 2=12]

12. Define group with example. Write any two properties of group. [1+1]

13. If $(G,*)$ is an abelian group prove that :

$(a * b)^{-1} = a^{-1} * b^{-1}$. [2]

14. Let G be the cyclic group of order n and suppose that a is generator of G then show that $a^k = e$ if and only if n divides k. [2]
15. Define field and field extension [2]
16. Prove that the intersection of two subrings of a ring is also ring. [2]
17. Define zero divisor with an example. [1+1]

The End