Mid-West University

Examinations Management Office

End Semester Examinations 2081

Bachelor level/ B.E. Computer/ 3rd Semester — Full Marks: 50

Time: 3 hours — Pass Marks: 25

Subject: Engineering Mathematics III (SH431/SH505)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a) i. Prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+1\right)$.

ii. Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing normal form. (2+3)

b) i. Show that every square matrix can be uniquely expressed as the sum of Symmetric and Skew-Symmetric matrices.

ii. Find the Eigen value and Eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$. (2+3)

2. a) i. Diagonalise the matrix $A = \begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}$.

ii. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$ along the curve $y = \sqrt{x}$ & $y = x^2$. (2+3)

b) State and prove the Green's theorem. (5)

3. a) i. Applying the Gauss' Divergence theorem to evaluate $\int\int_S \vec{F}.\hat{n}\, ds$ where, $\vec{F} = (2xy+z)\vec{i} + y^2\vec{j} - (x+3y)\vec{k}$ and V is the region bounded by the surface of the plane $2x+2y+z=6, x=0, y=0$ and $z=0$. (4+2)

ii. Find the inverse Laplace transform of $\log\left\{\frac{s(s+1)}{s^2+4}\right\}$.

b) i. Evaluate the integral $\int_0^\infty te^{-3t}\sin t\,dt$ by using Laplace transform.

ii. Using Laplace transform to solve $y'' + y' - 2y = x$ given $y(0) = 1, y'(0) = 0$. (2+3)

4. a) i. Prove that Second Shifting theorem, If $L^{-1}[F(s)] = f(t)$, then $L^{-1}[e^{-as}F(s)] = [f(t-a)u(t-a)]$. (3+2)

ii. Obtain the half range cosine series for $f(x) = (x^2)$ in range $0 \le x \le \pi$.

b) Obtain a Fourier series to represent $x + x^2$ form $x = -\pi$ to $x = \pi$ and deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ..$ (4)

5. a) Solve by Big-M method.

Maximize $z = 7x_1 + 4x_2$ Subject to $2x + y \le 150$, $4x + 3y \le 350$, $x + y \ge 80$ and $x, y \ge 0$. (5)

b) Solve by Simplex Method using duality:

Minimize $z = 21x_1 + 50x_2$

Subject to constraints $2x_1 + 5x_2 \ge 12$, $3x_1 + 7x_2 \ge 17$ and $x_1, x_2 \ge 0$. (5)

The End