Mid-West University

Examinations Management Office

End Semester Examinations 2081

Bachelor level/ B.E. Computer/ 4th Semester — Full Marks: 50

Time: 3 hours — Pass Marks: 25

Subject: Applied Mathematics (SH441/SH507)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a. i. Find the cube root of -1. [2.5]

ii. Show that the function $w = \log z$ is analytic everywhere except $z = 0$. [2.5]

b. Define Harmonic Function. Show that $u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$ is harmonic and find corresponding analytic function. [1+4]

2. a. i. Evaluate $\int_{1-i}^{2+i}(2x + iy + 1)dz$ along the straight line form $z = 1 - i$ *to* $z = 2 + i$. [2]

ii. Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$ where C is the circle $|z - i| = 2$. [3]

b. State Cauchy's Residues Theorem. Evaluate $\int_0^{2\pi}\frac{d\theta}{5-4\sin\theta}$ by contour integration in the complex plane. [1+4]

3. a. i. Prove that (Initial Valued Theorem): If $x(k) = 0$ for $k < 0$ *and* $Z[x(k)] = X(z)$ for $k \geq 0$, then $x(0) = \lim_{z\to\infty} X(z)$ provided the limit exist. [3]

ii. Find the Z-Transform of the function $X(t) = t^2e^{-at}$. [2]

b. Define Inverse Z-transform. Obtain the inverse z-transform of $X(x) = \frac{3z^3+2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

4. a. State Fourier Integral Theorem and show that the Fourier cosine integral representation of the function $f(x) = e^{-x}$ *is* $\int_0^\infty \frac{\cos\omega x}{1+\omega^2}d\omega = \frac{\pi}{2}e^{-x}$. [1+4]

b. i. Find the Fourier transform of $f(x) = \begin{matrix} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{matrix}$ and hence show that $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$. [3]

ii. Solve: $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial x} + u$ *where* $u(x,0) = 6e^{-3x}$. [4]

5. a. Drive the solution of Laplace equation in two dimension. [4]

b. Derive the one dimensional heat equation. [4]

The End