Mid-West University

Examinations Management Office

End Semester Examinations 2081

Bachelor level/ B.E. Civil/ 3rd Semester — Full Marks: 50

Time: 3 hours — Pass Marks: 25

Subject: Engineering mathematics III (SH431/SH203)

- *Attempt all the questions*
- *Figures in the margin indicate full marks.*
- *Assume suitable values, with a stipulation, if necessary.*
- *Candidates are required to answer the questions in their own words as far as possible.*

1. a) i) Find the rank of the matrix. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{bmatrix}$.

ii) State whenever the function is odd or even. Find the Fourier series representation of the periodic function $f(x) = k \quad If -\frac{\pi}{2} < x < \frac{\pi}{2}$

$= 0 \quad If \frac{\pi}{2} < x < \frac{3\pi}{2}$. **(2+3)**

b) i) Determine the vectors are linearly dependent or independent.

$x_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, x_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, x_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and $x_4 = \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}$.

ii) Find the characteristic equation of the matrix A = $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and hence find its inverse. **(2+3)**

2. a) i) Find the Laplace transform of the given function : $\frac{cos2t - cos3t}{t}$.

ii) State the convolution theorem of inverse Laplace transform. Find the inverse Laplace transform of $\frac{2as}{(s^2+a^2)^2}$. **(2+3)**

b) Solve the following differential equation by Laplace transform method.
$y''' + 2y'' - y' - 2y = 0,$ given that $y(0) = y'(0) = 0$ and $y''(0) = 6$. **(5)**

3. a) i) Evaluate: $\oint_C \vec{F}.\vec{dr}$ where $\vec{F} = x^2\vec{i} + y^3\vec{j}$ and C is the arc of the parabola $x = y^2$.

ii) Obtain the half-range sine series for $f(x) = e^{ax}$ *in the interval* $0 < x < \pi$. **(2+3)**

b) Solve the following equation by using Gauss elimination method.
$x + y - z = 3$, $2x - 3y + 9z = 60$ and $7x + 3y + 3z = 69$. **(5)**

4. a) State and proved Gauss Divergence Theorem.

OR

Verify Greens theorem for $\oint_C [(2xy - x^2)dx + (x + y^2)dy]$. Where C is the closed curve of the region bounded by $y = x^2$ and $x = y^2$. **(5)**

b) Verify Stokes theorem for the function $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$, Where S is the portion of the surface $x^2 + y^2 - 2ax + az = 0$ above the $xy - plane$. **(5)**

5. a) Define the Surface integral with example. Show that $\iint_S (4xz\vec{i} - y^2\vec{j} + yz\vec{k}).\hat{n}\, ds = \frac{3}{2}$ where, S is the surface of the cube x=0 ,x=1 , y=0 ,y = 1 , z =0 and z = 1 . **(1+4)**

b) Define the volume integral with example. Find the work done in moving partial in the force field
$\vec{F} = (3x - 4y + 2z)\vec{i} + (4x + 2y - 3z^2)\vec{j} + (2xz - 4y^2 + z^3)\vec{k}$, along one round of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$. **(1+4)**

The End