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Mid-West University

Examinations Management Office

Semester End Examination 2080

Bachelor level/ B. Sc. /4th Semester | Full Marks: 60

Time: 3 hours | Pass Marks:30

Subject: Linear Algebra I (MATH343/MTH443)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

Group – A [4x6 = 24]

1. Give definition and an example of REF and RREF.? Reduce the matrix A below to REF and

RREF: $A = \begin{bmatrix} 0 & -3 & 6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$

OR

What are the types of elementary row operations? What do you mean by basic variables and free variables? Show that the system:

$x + 2y - z = 3,\quad 3x - y + 2z = 1,\ 2x - 2y + 3z = 2,\ x - y + z = -1$

is a consistent and solve them reducing into row reduce echelon form.

2. Define symmetric and skew- symmetric matrix with an example. Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew- symmetric.

3. Define linear Transformation Kernel of linear Transformation. The linear map $x \to Ax$ be ingective if the corbel of A contains only one zero vector. Let $T: R^2 \to R$ be a linear transformation for which

$T(1,1) + 3,\ T(0,1) = -2$ than find the value of $T\left(1, \frac{1}{2}\right)$.

4. Let $L_1 and\ L_2$ be two line in R^n describe as

$L_1 = \{u + tv ; t \in R\} and\ L_2 = \{w + sz , s \in R\}$ Show that these lilne are same if and only if $u - w$ and v are multiple of z. Show that these two lines described parametrically R^3 are parallel and distinct:

$(2,3,-5) + t(1,-3,2)\ and\ (-1,-8,2) + s(-4,12,-8)$.

Group - B [6x4 = 24]

5. Define the linear dependent and independent vector of a vectors. determine the vector $(-1,3,7)$, $(4,2,7)$ and $(3,1,4)$ aer dependent and independent?

6. Is the vector $(4,5,1)$ in the span of the set consisting of $(3,5,-4), (2,1,-5)\ and\ (-2,1,3)$?

7. Define singular and non-singular matrix? Let $A = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ be the matrices of order 2. Determine the matrix B when A and B are inverse to each other.

8. Prove that in R^2 counterclockwise rotating of every point by an angle Ø is a linear transforming whose matrix$\begin{pmatrix} cos\emptyset & -sin\emptyset \\ sin\emptyset & cos\emptyset \end{pmatrix}$.

9. Define the rank of a matrix. Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

10. Given the domain and range of the determinant function.

    If Let A be the $n \times n$ order invertible matrix. Then show that the solution to the system of equation $Ax = b$ is given by these formula: $x_j = \left[\frac{Det A_j(b)}{Det(A)}\right] (1 \leq j \leq n)$

Group C [6x2 = 12]

11. Define consistent and inconsistent system of equations. Is the given system $-x + y = 1$ and $2x + y = 4$ consistent

12. Let weights 11kg, 2kg and 7kg be situated at these points (2, 1, 3), (1, -1 , 0), and (3 ,-2 ,1).Where should a weight of 5kg be situated so that the four weights would be in equilibrium? the weight are 3, 7 and 5 kilos respectively.

13. Define Null space. If $X = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and A = . $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, Is $X \in NulA$.

14. If x and y are the vectors in Ker(A) , and if α is a scalar then x + y and αx are also in Ker(A).

15. Let S be a triangle or parallelogram having vertex at 0 and A be the matrix. Let$T: R^2 \rightarrow R^2$ be a linear mapping define by $T(X) = AX$ then show that $Aera[T(S)] = |Det(A)|Area(S)$.

16. Us the crammer's rule to solve the system: $x + y + z = 9,\ 2x + 5y + 7z = 52$ *and* $2x + y - z = 0$.

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