Mid-West University

Examinations Management Office

End Semester Examinations -2080

Bachelor level/ B. Sc /6th Semester
Time: 3 hours
Subject: Discrete Mathematics (Math461)

Full Marks: 100
Pass Marks: 50

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

GROUP A [6(2+2)=24]

1) a) Define even number. Prove that for all natural numbers n, $n^2 + n$ is an even number.

b) Define odd number. Show that 17 is a prime number.

2) a) Let D_n denote natural number that divide n then find $D_{60} \cup D_{75}$ and $D_{60} \cap D_{75}$.

b) Show that $\sim p \vee q$ and $p \Rightarrow q$ are logically equivalent.

3) a) Prove that a natural number is a multiple of 12 if and only if it is a multiple of 3 and a multiple of 4.

b) For any $a, b, c \in \mathbf{N}$, if $a < b$ then $a + c < b + c$.

4) a) The formula $f(n) = 2n$ defines a function $f: N \to N$. Prove that f is injection but not surjection.

b) Prove that if $X \subseteq Y$ and X is infinite then Y is infinite.

5) a) Find the positive integer r and s such that $\frac{r}{s}$ is equal to $0.100\overline{17}$.

b) For any $x, y, z \in \mathbb{Z}$, prove that $x - (y - z) = (x - y) + z$.

6) a) Show that $\sqrt{3}$ is irrational number.

b) Show that $\binom{n}{k} = \binom{n}{n-k}$ $for\ 0 \leq k \leq n$.

GROUP B [13x4=52]

7) Prove that there is no greatest natural number. For any natural number n, $n^2 + 7n + 12$ is an even number.

8) Define Conjuction and disjunction. Construct the truth table of $p \Rightarrow \sim q \wedge \sim r$

9) Use method of induction show that, $\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5)$

10) Define least member of $X \subseteq \mathbf{N}$. If subsets of X_1, X_2 of **N** has least member then $X_1 \cup X_2$ and $X_1 \cap X_2$ have least member.

11) Define injection and surjection functions. A function $g: \mathbf{N} \to \mathbf{N}$ defined by $g(g(x)) = x$. Prove that g is bijection. but not surjection.

12) If $x \backslash 0$ for all $x \in Z$ but $0 \backslash x$ only when $x = 0$ and, if p and p' are prime and $p \backslash p'$, then, prove $p = p'$.

13) Define cardinality of a set. Let m be a natural number, then every natural number n, there is an injection from N_n to N_m then $n \leq m$.

14) Show that if $c \backslash a\ and\ c \backslash b$, then $c \backslash xa + yb$ for any integer x, y. If x and y are non-zero integers such that $x \backslash y\ and\ y \backslash x$ then either $x = y\ or\ x = -y$.

15) Define countable set. Prove that set of real number is not countable.

OR

Prove that the set of rational number is countable.

16) Let S_n be set of permutations of $\{1, 2, \ldots, n\}$, then (i) if $\pi, \sigma \in S_n$ then $\pi\sigma \in S_n$.

(ii) if $i \in S_n$ be identity function then for $\sigma \in S_n$ $i\sigma = \sigma i = \sigma$.

17) For any positive integers m and n, prove that

$$\binom{m-1}{0} + \binom{m}{1} + \cdots + \binom{m+n-2}{n-1} + \binom{m+n-1}{n} = \binom{m+n}{n}.$$

OR

Define Mobius function. Let g be a function defined on **N** and function f defined as $f(n) = \sum_{d \backslash n} g(d)$ then $g(n) = \sum_{d \backslash n} \mu(d) f\left(\frac{n}{d}\right)$.

18) Define partition of a set. Let $S(n, k)$ be partition of a n-set X into k parts, show

that $S(n, 2) = 2^{n-1} - 1$

19) Define invertible element of $\mathbb{Z}_m$. If y is invertible in $\mathbb{Z}_m$ then $y^{\varphi(m)} = 1$ in $\mathbb{Z}_m$.

GROUP C [4x6=24]

20) Define principle of induction. Prove that if $x_1 = 3, x_2 = 5$, $x_{n+1} = 3x_n - 2x_{n-1}, \forall n \geq 2, then\ x_n = 2^n + 1, \forall n \in \mathbf{N}$.

21) If p is a prime and $x_1, x_2, \ldots, x_n$ are any integers such that $\frac{p}{x_1 x_2 \ldots\ldots x_n}$,