Mid-West University

Examinations Management Office

End Semester Examinations -2080

Bachelor level/ B. Sc /6th Semester | Full Marks: 100
Time: 3 hours | Pass Marks: 50

Subject: Modern Algebra I (Math463)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

GROUP A [6(2+2)=24]

1. a) State the Euclidean Algorithm of integer
   b) Define binary operation. Is the binary operation $*$ define on $\mathbb{Z}$ letting by $a * b = a - b$ satisfied the commutative and associative properties
2. a) Define Abelian group. If $(G,*)$ is an abelian group prove that : $(a * b)^{-1} = b^{-1} * a^{-1}$.
   b) Define Normal subgroup with an example.
3. a) Define cosets of a group. Exhibit the left cosets $3\mathbb{Z}$ of $\mathbb{Z}$
   b) Define terms Normalizes and Centralizers.
4. a) Let $(\mathbb{R},+)$ and $(\mathbb{R}', \ .)$ be two groups and $\emptyset : \mathbb{R} \to \mathbb{R}'$ be a mapping define by $\emptyset(x) = e^x, x \in \mathbb{R}$. Show that $\emptyset$ is homomorphism.
   b) Let a be an element of order 2 in group G. Show that the collection $H = \langle a : a^2 = 1 \rangle$ is a normal in G iff $a \in Z(G)$.
5. a) Define zero divisor with an example.
   b) If f is homomorphism of ring $\mathbb{R}$ into the ring $\mathbb{R}'$ then show that $f(0) = 0'$, where 0 is a zero element of $\mathbb{R}$ and $0'$ is a zero a zero element of $\mathbb{R}'$.
6. a) Define field and field extension.
   b) Prove that the intersection of two subrings of a ring is also ring.

GROUP B [13x4=52]

7. Define group. Show that the set $G = \{a + b\sqrt{2} : a, b \in G\}$ is a group under multiplication.
8. Show that the intersection of two subgroup of a group is group. What about their union?
9. Define cyclic group. Show that every subgroup of cyclic group is cyclic.
10. Define homomorphism. Let G and H are two subgroups and $\varphi : G \to H$ is homomorphism , then show that $ker\varphi$ is normal subgroup.
11. Define center of group. Let G be a group and X be a nonempty subset of G then prove that $Z(G) \subseteq C_G(X) \subseteq N_G(X) \subseteq G$.
12. Define Epimorphism of a group G. Let $(\mathbb{Z}, +)$ and $(\{1, -1\}, \ .)$ be a two groups. Define a mapping $\varphi : \mathbb{Z} \to \{1, -1\}$ by

    $\varphi(X) = 1$, if n is even

    $\varphi(X) = -1$, if n is odd

    Prove that φ is Epimorphism.
13. Define Cayley Table. Let $G = \{1, -1, i, -i\}$ draw Cayley Table. Also define Klien four group with their properties.
14. Let G be a cyclic group of order n. then show that G contain one and only on subgroup of order d iff d/n .

    OR

    Prove that a group of order n is cyclic iff it has an element of order n.
15. Show that every finite group G is isomorphic to a permutation group.
16. Define Factor group. A homomorphism $f : G \to G'$ with $kerf$ is an isomorphism of G into G' then prove $K_f = \{e\}$.
17. Prove that every quotient ring is a isomorphic image of ring R
18. Define integral domain. Prove that every field in integral domain.
19. Define commutative ring. Prove that the finite commutative ring without zer divisor is a field.

    OR

    Define subring of a ring. A non-empty subset S of a ring is a sub ring of R iff for $a, b \in S$ then i) $a - b \in S$ ii) $ab \in S$.

GROUP C [4x6=24]

20. Let G be a cyclic group of generated be a,
    a) If G is finite order n then an element $a^k \in G$ is a generator of G iff k and n are relatively prime.
    b) If G is a finite order then a and a^{-1} are the only generator of G.

21. Define index of subgroup. Stare and prove the Lagrange's Theorem. Let $(\mathbb{R},+)$ and $(\mathbb{R}^+,\ .)$ be a two groups prove that the mapping $\emptyset : \mathbb{R}^+ \rightarrow \mathbb{R}$ define by $\emptyset(x) = logx$ is a isomorphism.

22. Define isomorphism of a group and factor group. Let H be the normal subgroup of group G and $\varphi : G \rightarrow \frac{G}{H}$ is a mapping define by $\varphi(a) = aH, \forall a \in G$, then show that φ is isomorphism and $ker\varphi = H$.

23. If S is an ideal of ring R then the set $\frac{R}{S} = \{S + a : a \in R\}$of all residue class of S in R form a ring for the two composition define as

a) $(a+S)+(b+S)=(a+b)+S$ b) $(a+S)(b+S)=(ab)+S$.

OR

Define maximal ideal of a
ring. An ideal S of a ring of integers I is a maximal if S is generated by some prime integers.

THE END